SYNCHROPHASOR AND FREQUENCY ESTIMATION ALGORITHM

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Abstract: In this paper, a new algorithm for synchrophasor and frequency estimation in power systems is proposed. The proposed approach is based on transformation of the measured signal into a complex sinusoid by using the Constant Modulus algorithm. The transformed signal and transformation coefficients are used for estimation of the parameters of interest. Simulation results show that the proposed algorithm exhibits better performances compared to the considered algorithms.

1. INTRODUCTION

Phasor measurement units (PMUs) are new devices aimed at measuring phasors and frequencies of current and voltage signals in power systems [1]. All of the measurements are time-synchronized with the coordinated universal time (UTC) using the satellite global position system (GPS), [1]. The IEEE standard C37.118.1-2011 describes the performance requirements that PMU devices need to implement, [2]. It divides PMU devices into two groups: a) P class, faster and less accurate; and b) M class, slower and more accurate. P class devices target the usage in system protection applications, while M class devices find use in power system monitoring, [2].

Traditional synchrophasor estimation algorithms rely on the concept of steady state measurements. It is assumed that the phasor parameters do not change within the time windows in which the estimation is performed. Among those algorithms, the most popular ones are based on Discrete Fourier Transform (DFT) due to its good filter characteristics and robustness even with presence of higher harmonics the signal. However, in cases when the fundamental frequency varies, the DFR based methods experience performance degradation. Therefore, methods that calculate phasor as a function of estimated fundamental frequency are used, [3]. Newton method and Kalman filters also find application in synchrophasor parameters estimation, [4, 5].

The above mentioned methods are designed to provide good results for steady state conditions. Latest IEEE standard defined accuracy requirements pertinent to transient process, i.e. for step changes od amplitude or phase in current and voltage signals. That

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prompted development of number of algorithms for dynamic phasor estimation. These algorithms provide better performance in dynamic conditions, i.e. transients, than the conventional methods, [6, 7]. Most of proposed algorithms are numerically complex and may introduced delays due to calculation times. In [8], the authors propose an iterative Least Square Error (LES) method, which is simple and yet conforms to the requirements of the latest IEEE standard.

This paper proposes a new approach for estimating synchrophasors and frequency based on transforming the real sinusoid signal into a complex one using Constant Modulus Least Square algorithm (LSCMA). The proposed algorithm complies with all of the IEEE C37.118.1-2011 requirements. Simulation results shows that the proposed LSCMA method in most test scenarios exhibits better performance than the considered algorithms.

The paper is organized as follows. After Introduction, the model for synchrophasor estimation is given, and description of estimation algorithms used for comparison is provided. Next section provides details on the proposed algorithm. Simulation results and Conclusions are provided at the end.

2. SYNCHROPHASOR AND FREQUENCY ESTIMATION

In power system analysis, the sinusoid voltage and current signals are represented using phasors. A signal with amplitude X_m, frequency ω, and phase ϕ described by

$$x(t) = X_m \cos(\omega t + \phi) \tag{1}$$

is represented by its phasor as:

$$X = (X_m / \sqrt{2}) e^{j\phi} = X_r + jX_i, \tag{2}$$

where X_r and X_i represent real and imaginary part of the phasor. Synchrophasor representation of the signal (1) is the value of X in (2), where ϕ represents instantaneous phase shift related to a cosine function of nominal frequency, which is synchronized with the UTC, [2].

The LES method described in [8] assumes that the measured electrical signal contains first N harmonics:

$$x(nT) = \sum_{n=1}^{M} x_n \cos(2\pi nTf + \theta_n) + \sigma(nT), \tag{3}$$

where n corresponds to discrete time, T is the sampling period and $\sigma(nT)$ is white Gaussian noise.

Real and imaginary parts of such synchrophasor are estimated as follows, [8]:

$$\mathbf{x}(i) = \mathbf{A}^{\dagger}\mathbf{y}(i). \tag{4}$$

Vector $\mathbf{y}(i)$ corresponds to measured values of the voltage signals within i-th window having a length of M, while $\mathbf{A}$ matrix has dimension $N \times M$, and its elements are given by $\cos(n2\pi f_{i-1} mT)$ and $\sin(n2\pi f_{i-1} mT)$ for n=1,...,N, and m=-M/2,...,M/2, [8]. First two elements of vector $\mathbf{x}$ correspond to real and imaginary parts of the synchrophasor that needs to be reported for the considered i-th time window, [8]:

$$\hat{X}_r(i) = x_1(i), \hat{X}_i(i) = x_2(i). \tag{5}$$

The fundamental frequency component is estimated using, [8]:

$$\hat{f}(i) = \left(\tan^{-1}\frac{\hat{X}_i(i)}{\hat{X}_r(i)} - \tan^{-1}\frac{\hat{X}_i(i-1)}{\hat{X}_r(i-1)} \right) / \left(2\pi TN\right). \tag{6}$$

Instead of the instantaneous frequency, the LES estimation method uses the frequency estimated in the previous step (time windows), and then, based on the estimated parameters it calculates the frequency for the current time windows. If there is a significant difference between estimated frequencies $\hat{f}(i)$ and $\hat{f}(i-1)$, the calculation iterations are repeated predefined number of times or until the difference between the frequencies estimated in two steps falls under the desired accuracy.

In the IEEE standards, a dynamic Finite Impulse Response (FIR) filter based on a DFT is given as a reference algorithm to estimate synchrophasors, [2]. Synchrophasor parameters are estimated as:

$$\hat{X}(i) = \frac{2}{G}\sum_{k=-M/2}^{M/2} x(i+k)\times W(k)\times \exp(-j(i+k)T\omega_0). \tag{7}$$

where $W(k)$ represents coefficients of the low-pass filtar:

$$W(k) = \frac{\sin(4\pi F_{fr}Tk)}{4\pi F_{fr}Tk}h(k), \tag{8}$$

where k=-M/2,…,M/2, F_{tr} is the average frequency of the low-pass filter, T sampling period, $h(k)$ Hamming window, and G gain obtained by summing the coefficients $W(k)$:

$$G = \sum_{k=-M/2}^{M/2} W(k). \tag{9}$$

3. PROPOSED ALGORITHM

For an electric signal given in (1), there exists a pair complex coefficients that satisfy the following:

$$r(nT) = e^{j\omega nT} = W_1 x(nT) + jW_2 x(nT-\Delta), \tag{10}$$

where Δ represents a delay equal to a multiplication product of an integer and the sampling period T. It can be shown that W_1 and W_2 satisfy:

$$W_1 = \frac{e^{-j\omega\Delta}}{A\sin\omega\Delta},\ W_2 = \frac{j}{A\sin\omega\Delta}. \tag{11}$$

For the i-th window of M samples, the coefficients W_1 and W_2 can be determined by minimizing the performance function:

$$J(i) = \left|1 - \mathbf{W}^H(i)\mathbf{X}(i)\right|, \tag{12}$$

where vector **W** contains coefficients W_1 and W_2, and $\mathbf{X}(i)$ is a matrix of dimension $M\times 2$, which contains elements $x(nT+k)$ and $x(nT-\Delta+k)$, for $k=-M/2,\ldots,M/2$. Vector **W** can be iteratively updated using Least Square Constant Modulus algorithm (LSCMA):

$$\mathbf{W}(i) = \lambda \mathbf{W}(i-1) + (1-\lambda)\mathbf{X}^{\dagger} y(i), \tag{13}$$

where λ is forgetting factor and should be set close to 1.

In every window i, the amplitude of the synchrophasor is calculated as:

$$\hat{X}_m(i) = \frac{\sin(\arg\{W_2(i)/W_1(i)\})}{|W_1(i)|}. \tag{14}$$

while the frequency is estimated as:

$$\hat{f}(i) = |\pi/2 + \arg\{W_2(i)/\mathrm{W}_1\}| / (2\pi\Delta). \tag{15}$$

Finally, the instantaneous phase is estimated as:

$$\phi(i)=\arg\{W_2(i)r(i)\}, \tag{16}$$

where

$r(n)$ is the transformed signal:

$$r(i) = \mathbf{W}(i)^H \mathbf{X}(i). \tag{17}$$

Having in mind that electric signals may contain higher harmonics, the input signal x, before processing with LSCMA, needs to be preprocessed using a low-pass filter. The use of filter introduces gains/attenuation of the amplitude and phase shift. That is the reason to introduce an amplitude and phase correction of the transformed signal:

$$r(i) = \mathbf{W}(i)^H \mathbf{X}(i) A_F(f_i) e^{j\phi_F(i)}, \tag{18}$$

where $A_F(f_i)$ and $\phi_F(i)$ are amplitude and phase characteristics of the low-pass filter at $f(i)$ frequency. These correction coefficients are changed only if there is a change in the fundamental frequency component. It is desired that the low-pass filter exhibits a flat amplitude characteristics and has a constant group delay in frequency range of interest (±5Hz of the nominal frequency, [2]).

4. SIMULATION RESULTS

For the simulation, the nominal frequency of the fundamental component is set to 50Hz. The sampling rate is set to 1250Hz, and reporting frequency is 25 frames/s. Time window widths for the estimation using LSCMA, LES, and DFT methods are 25, 50, and 258 samples, respectively. The Δ parameter for LSCMA is set to $5T$. Total vector error (TVE) and deviation from the actual frequency (DF) measures are used as the performance measures, [2]. Various test scenarios have been implemented. Maximal values for TVE and

DF given by the standards for each of the scenarios are listed in the headers of tables showing simulation results.

A. STEADY STATE TESTS

Table 1.a shows maximal values of TVE in considered algorithms with respect to different values of the fundamental frequency. It can be noted that LSCMA and LES algorithms display no error when estimating synchrophasors, while DFT exhibits small error that grows with deviation of the fundamental frequency. Similar results can be observed for the estimation of frequency (Table 1.b).

Table 1.a TVE in considered algorithms with respect to different values of the fundamental frequency

	Max. TVE (1%)				
f [Hz]	45	47	50	53	55
LSCMA	0	0	0	0	0
LES	0	0	0	0	0
DFT	0.34	0.29	0.048	0.25	0.34

Table 1.b DF in considered algorithms with respect to different values of the fundamental frequency

	DF [0.005Hz]				
f [Hz]	45	47	50	53	55
LSCMA	0	0	0	0	0
LES	0	0	0	0	0
DFT	0.002	3×10^{-4}	0	3×10^{-4}	0.003

Simulation results, when the electric signal contains a random harmonic with an amplitude of 10% of the amplitude of the fundamental components, are shown in Table 2.a and 2.b. In this case, LES method exhibits the smallest TVE due to modeling of harmonics. However, if a good performance is needed for all 50 harmonics, LES method becomes numerically complex since for every frame there is a need to calculate a pseudoinverse matrix with dimension of a 25×100. When using the proposed algorithm, TVE drops when increasing a number of harmonics, due to the use of Butterworth filter in which the gain drops for higher frequencies. For all harmonics, DFT exhibits a second order error. As for the frequency estimation, all three algorithms gave good results.

Tabela 2.a TVE in considered algorithms with respect to occurrence of different harmonics

	Max. TVE (%)				
Harmonic	1	2	3	7	15
LSCMA	0.036	7×10^{-3}	1×10^{-3}	1×10^{-7}	7×10^{-8}
LES	0	0	0	0	0
DFT	0.034	0.085	0.075	0.06	0.048

Tabela 2.b DF in considered algorithms with respect to occurrence of different harmonics

Harmonic	DF (0.025Hz)				
	1	2	3	7	15
LSCMA	10^{-13}	10^{-13}	10^{-13}	10^{-13}	10^{-13}
LES	0	0	0	0	0
DFT	0	0	0	0	0

B. DYNAMIC TESTS

The accuracy of the synchrophasor algorithms for amplitude and phase modulated signals is defined in the IEEE standard as well. Tables 3.a and 3.b show simulation results for test cases where the phase changes $1+0.1\cos\omega t$, and various frequencies ω have been considered. It can be seen that the proposed agorithm display best results with respect to TVE and DF. LES shows similar performance when estimating synchrophasors (TVE), while the DF error is multiple times higher than in the proposed algorithm. On the other hand, the DFT algorithm has the opposite behaviour when compared to LES.

Table 3.a TVE in considered algorithms with respect to phase modulated test signals

f [Hz]	Max. TVE (3%)				
	0.1	1	2	3.5	5
LSCMA	1×10^{-5}	1×10^{-3}	0.005	0.02	0.04
LES	6×10^{-5}	0.005	0.02	0.06	0.14
DFT	0.048	0.048	0.049	0.05	0.051

Tabela 3.b DF in considered algorithms with respect to phase modulated test signals

f [Hz]	DF TVE (0.3Hz)				
	0.1	1	2	3.5	5
LSCMA	10^{-6}	0.0003	0.001	0.004	0.003
LES	0.08	0.1	0.12	0.15	0.005
DFT	2×10^{-6}	0.0004	0.0015	0.005	0.009

Simulation results with respect to amplitude modulation when amplitude of the fundamental component follows $1+0.1\cos\omega t$ law are shown in Tables 4.a and 4.b. In this set of tests, the proposed LSCMA exhibited the best accuracy, while LES method was the worst.

Table 4.a TVE in considered algorithms with respect to amplitude modulated signals

	Max. TVE (3%)				
f [Hz]	0.1	1	2	3.5	5
LSCMA	1×10^{-5}	0.0003	0.0006	0.001	0.002
LES	0.008	0.08	0.17	0.29	0.4
DFT	0.048	0.048	0.049	0.05	0.052

Table 4.b DF in considered algorithms with respect to amplitude modulated signals

	DF (0.3Hz)				
f [Hz]	0.1	1	2	3.5	5
LSCMA	0.0002	0.0015	0.003	0.055	0.078
LES	0.008	0.08	0.17	0.29	0.4
DFT	0.02	0.042	0.06	0.075	0.1

Tables 5.a and 5.b show simulation results when frequency of the fundamental component continuously rise (linear law). In this set of test cases, the proposed LSCMA method displayed the best performances when estimating synchrophasors and freqyency.

Table 5.a Comparison of considered algorithms when the frequency continuously rise (TVE)

Ramp	TVE (1%)				
slope	0.1	0.2	0.5	0.8	1
LSCMA	0.0008	0.006	0.016	0.025	0.035
LES	0.015	0.08	0.08	0.1	0.15
DFT	0.35	0.35	0.35	0.35	0.35

Table 5.b Comparison of considered algorithms when the frequency continuously rise (DF)

Ramp	DF (0.05Hz)				
slope	0.1	0.2	0.5	0.8	1
LSCMA	0.0006	0.001	0.002	0.003	0.004
LES	0.002	0.004	0.01	0.015	0.04
DFT	0.005	0.008	0.011	0.02	0.03

Table 6 Comparison of considered algorithms when there is a step change in the amplitude of test signals

	Amplitude			Freq.
10% step	Π (10%)	T_d (0.01s)	T_r (0.28s)	T_r (0.56s)
LSCMA	0.35%	0.002s	0.028s	0.0328s
LES	0.10%	0.0096s	0.022s	0.0240s
DFT	0.83%	0.006s	0.021s	0.0190s

Synchrophasor estimation algorithms should also provide satisfactory response in cases of sudden or step changes in input signal amplitude. The following characteristics of the response are considered in such cases: overshoot (Π), time delay (T_d) and response time (T_r), [2]. Table 6 shows test results for considered algorithms. It can be noted that all of the teted algorithms exhibited satisfactory results as per IEEE standard. LES exhibited the smallest overshoot, LSCMA smallest time delay, and DFT shortest response time.

5. CONCLUSIONS

This paper describes a new algorithm for synchrophasor and frequency estimation in power systems. The proposed algorithm is based on transforing the real sinusoid signal into a complex one using LSCMA algorithm. The transform coefficients are directly used to estimate the amplitude and frequency of the input signal. The transformed complex signal is used to estimate the phase. Low-pass filtering of the input signals is needed prior to the application of the proposed method in order to reduce effects of higher harmonics. The proposed algorithm has been tested using simulation of various input test signals. The test results show that the method satisfies all of the IEEE IEEE Std. C37.118.1-2011 requirements. Simulation results also showed that the LSCMA algorithm shower better performance in majority of test scenarios comparing to the considered algorithms.

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