CHAOTIC DYNAMICS IN HELICOPTERS VIBRATIONS

Vesna Rubežić, Luka Lazović**, Marko Babić*** and Ana Jovanović*****

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Abstract: In this paper, the linear dynamic behavior of the helicopter in terms of characterization of the vibrations in a helicopter, is described. After analyzing in the time and frequency domain, vibrations are analyzed in geometrical and topological methods of chaos theory. Although the vibration in a helicopter are mainly periodic, the presence of chaotic dynamics using a systematic and detailed approach, is investigated. The aim of the research is to characterize the vibration mechanism based on a one set of measured data.

1. INTRODUCTION

The helicopter is a complex nonlinear dynamic system [1]. Under certain conditions, the nonlinear dynamic system can exhibit chaotic behavior. The basic characteristics of chaos are a great sensitivity to initial conditions, noise similar signal in time domain and broadband power spectrum of chaotic signal [2] - [5].

The theory of vibrations on the helicopter, theoretical analysis of vibration signals and vibration control are the current topics of research for many years [6], [7], [8], [9]. Reduction and control of vibrations are one of the challenges in the modern development of this group of aircraft. This issue, therefore, is open, the current, challenging and attractive for researchers. It is necessary, first of all, to accurately characterize the vibration signal. The vibrations of the helicopter have generally periodic nature. However, in some vibrations were found the elements of chaotic dynamics [9].

This paper analyzes the measured signal of vertical vibration of the rail beneath the pilot's seat in a helicopter Gazela. Applying classic, geometric and topological methods of analysis the chaotic dynamics of this signal is determined.

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* Vesna Rubežić is with the Faculty of Electrical Engineering, University of Montenegro, Montenegro (e-mail: vesnar@ac.me).

** Luka Lazović is with the Faculty of Electrical Engineering, University of Montenegro, Montenegro (e-mail: lukal@ac.me).

*** Marko Babić is with Montenegro Airlines, Podgorica, Montenegro (maka1423@gmail.com)

**** Ana Jovanović is with the Faculty of Electrical Engineering, University of Montenegro, Montenegro (e-mail: anaj@ac.me).

The paper is organized as follows. After the introduction, section 2 provides a brief description of the source of vibration in a helicopter. In section 3 the main indicators of chaos are given and briefly described. In section 4, classical, geometric and topological methods of chaos theory have been applied to a set of data obtained by measuring the vibration on the helicopter. Finally, conclusions are given.

2. VIBRATIONS ON HELICOPTER

Gazela helicopter is a light transport helicopter of second generation (Figure 1). The main sources of vibration in a helicopter are: the main rotor, engine and tail rotor. Rotor works in the complex aerodynamic field. Aerodynamic loads on the rotor blade varies significantly in their movement around the axis of rotation. In the case stationary flight aerodynamics loads are periodic.
Changes in the aerodynamic flow conditions are the causes of vibrations on the helicopter, which are transmitted via the connecting elements to the structure of the helicopter, i.e. to the area of the crew, passengers or cargo area [1], [8], [9].

Figure 1 Helicopter Gazela

Measurement and analysis of vibration levels on individual elements of the structure of the helicopter are used for the development of elements for vibration damping.
Considering the continuous tightening of quality requirements in the world standards which define the levels of human and structural vibration, measurement and analysis of vibrations on the helicopter act as initiators for development and modification of the systems.

3. KEY INDICATORS OF CHAOS

Chaos is present only in certain deterministic systems. Deterministic system which sensitive to changes in initial conditions is defined as chaotic. Chaotic motion looks like random, Fourier spectrum is continuous and a broadband. Distinguishing between a chaotic and random signals from the experimental observation is a demanding task. In order to determine the chaos in nonlinear dynamic system, is necessary to do the following steps [2], [3], [5]:

- observe the signal in the time domain
- calculate the Fourier spectrum of the signal
- construct the phase portrait in the phase plane and / or space
- construct Poincare sections in phase space
- calculate the largest Lyapunov exponent
- calculate the fractal dimension

The first step in chaos determination is observing the signals in time domain. The motion, which is not periodically is chaotic or random. Fourier spectrum of the chaotic signal is continuous and noise similar. For the estimation of power spectral density of the chaotic signals in this paper is used periodogram.

Assuming that the signal has N samples, where the time between two samples is Δt and a value x_n, periodogram is defined as the square of the module of a discrete Fourier transform [3]:

$$S(f) = \frac{\Delta t}{N}\left|\sum_{n=0}^{N-1} x_n e^{-i2\pi nf}\right|^2, \qquad -\frac{1}{2\Delta t} < f \leq \frac{1}{2\Delta t} \tag{1}$$

Where the $1/2\Delta t$ represents Nyquist frequency. The difference between chaotic and random signals can't be determined only with Fourier spectrum. Information that leads to the discovery of non-linear dynamics can be obtained by constructing the phase portraits in a two-dimensional (2D) phase plane or three-dimensional (3D) phase space. In order to construct a 2D or 3D plane phase space we need two or three variables that describe the dynamics of the system. Only one variable is measured usually such as amplitude or acceleration. Takens [10] introduced a method by which from one observable variable we obtain "false" variable necessary to reconstruct the dynamics of the system. Let's observable variable be $x(t)$. Then, the second variable, $y(t)$, is formed by moving the time series $x(t)$ for τ, while the third variable, $z(t)$, moving the time series $x(t)$ for 2τ. This movement is called "embedding" time. In our simulations, "embedding" time is $\tau = 10$. Mathematically written, the process can be described as:

$$y(t) = x(t+\tau) \tag{2}$$

$$z(t) = x(t+2\tau) \tag{3}$$

In this way the pseudophase space is obtained. Attractor constructed in such space is equivalent to a real unknown attractor system. Details about "embedding" time can be found in [10]. The chaotic signal forms the trajectory that does not repeat, does not close and does not fulfills the entire phase space. Trajectory of random signal fulfills phase space entirely. Next step is to consider the concept of Poincare's sections (PS). One method for the construction of the Poincare sections is to set 2D surface in 3D space and observation of phase points where the trajectory passes through this surface. The chaotic signal gives the intersects which are limited to a small area, while random signal intersections fills the whole plane. Those geometrical and topological methods are qualitative indicators of chaos. One of the quantitative tests for chaos detection is the computation of Lyapunov exponents [11]. The largest Lyapunov exponents measures the speed of exponential separation of the two trajectory that was started from similar initial conditions [5], [11]. Quantitatively, the two trajectories in phase space with an initial spacing $\delta\mathbf{Z}_0$ diverges with a rate that can be expressed as:

$$|\delta\mathbf{Z}(t)| \approx e^{\lambda t}\,|\delta\mathbf{Z}_0| \tag{4}$$

where λ represents Lyapunov exponent. There are as many exponents as dimensions in phase space and they constitute Lyapunov spectrum of the dynamic system.
Chaotic system has at least one Lyapunov exponent greater than zero. Maximum Lyapunov exponent defines the predictability of dynamic systems. It is defined as:

$$\lambda = \lim_{t\to\infty}\lim_{\delta\mathbf{Z}\to 0}\frac{1}{t}\ln\frac{|\delta\mathbf{Z}(t)|}{|\delta\mathbf{Z}_0|} \tag{5}$$

Another quantitative test for chaos is fractal dimension. One of the most usable is Higuchi's fractal dimension [12], [13]. Fractal dimension of chaotic signal should be between 1 and 2, because a simple curve has dimension equal 1 and a plane has dimension equal 2. Higuchi's algorithm calculates fractal dimension of a time series in the time domain. Higuchi's algorithm, from a given time series, *X(1),X(2),..., X(N)*, constructs *k* new time series:

$$X_{km}\ :\ X(m), X(m+k), X(m+2k),\ \ldots, X\left(m+\operatorname{int}\frac{N-m}{k}k\right), for \quad m=1,2,...,k \tag{6}$$

Where *m* is initial time, *k* is interval time, *int(r)* is integer part of a real number *r*.
The "length" $L_m(k)$ of each curve X_{km} is then calculated as

$$L_m = \frac{1}{k}\left(\sum_{i=1}^{\operatorname{int}((N-m)/k)}|X(m+i\cdot k)-X(m+(i-1)\cdot k)|\right)\times\frac{N-1}{\operatorname{int}\left(\frac{N-m}{k}\right)\cdot k} \tag{7}$$

where *N* is total number of samples.
The "length" of curve for the time interval *k*, *L(k),* is calculated as the mean of the *k* values $L_m(k)$ for $m = 1, 2,..,k$:

$$L(k) = \frac{\sum_{m=1}^{k} L_m(k)}{k} \tag{8}$$

The value of fractal dimension, D_f, is calculated according to the following formulae:

$$D_f = \frac{n\sum(x_k\cdot y_k)-\sum x_k\sum y_k}{n\sum x_k^2-\left(\sum x_k\right)^2} \tag{9}$$

where $y_k = \ln L(k)$, $x(k) = \ln(1/k)$.
Determination of the presence of chaos from the analysis of one set of available data is a complex task. No one approach alone does not provide a definite confirmation of chaos. Only the use of more available techniques can lead to a reliable determination of the presence of chaos in the studied system.

4. THE RESULTS OF MEASURED DATA ANALYSIS

The paper presents the results of in-flight measurements in Gazela helicopter with exactly the required profile of the flight, at a height of $H=1000\ m$. The measured signal of vertical vibrations on the rail below the pilot's seat in a helicopter is analyzed. Figure 2 shows the time history of vibration during flight. Figure 3 shows the observed signal periodogram.

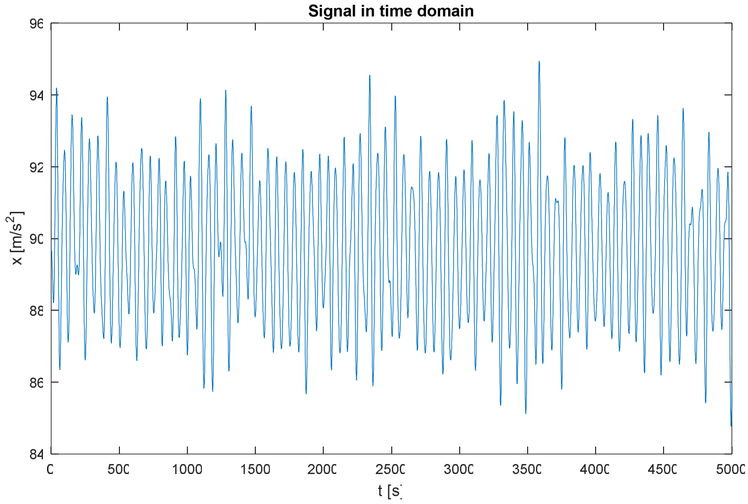

Figure 2 The time history of vibration during flight

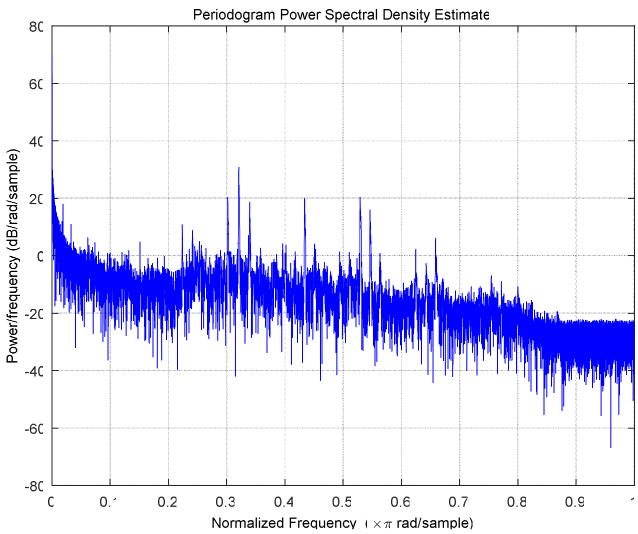

Figure 3. Periodogram of observed signal.

Aperiodic nature of the signals in the time domain is reflected in a noise similar broadband power spectrum.

Reconstructed attractor in pseudophase space is shown on Figure 4. It can be seen that the signal forms orbit that does not repeat, does not close and do not fulfill the entire phase space. Poincare sections are shown in Figure 5. It can be seen that the cross-sections appear as expected for a chaotic signal.

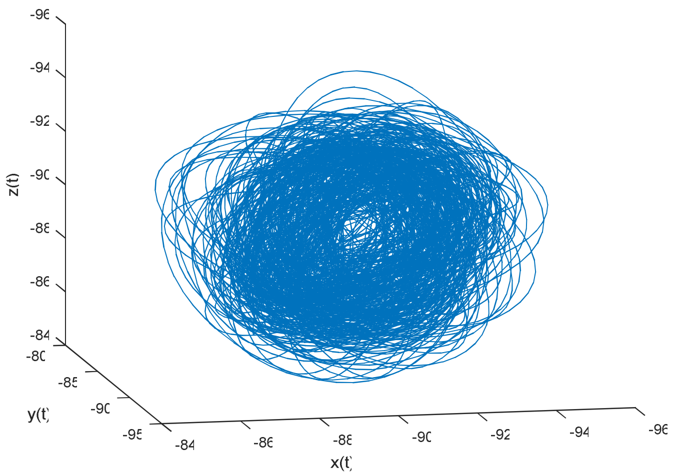

Figure 4. Trajectory in three-dimensional pseudophase space.

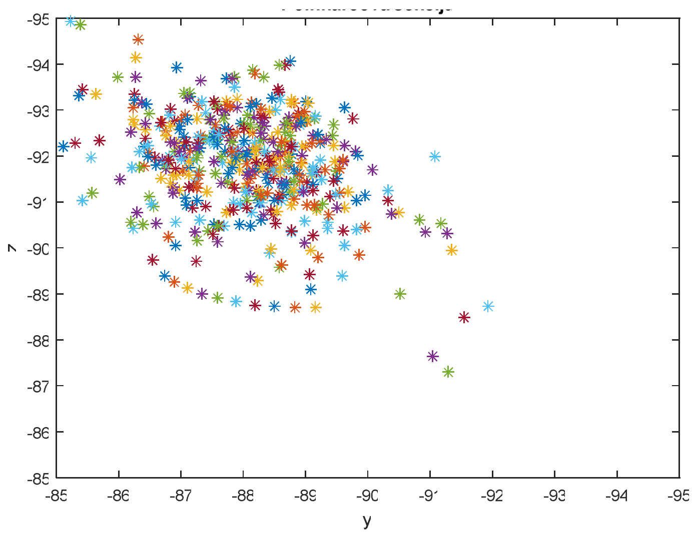

Figure 5. Poincare section of the 3D phase space. Section is made of points from trajectory from Figure 4 passing through the plane x = -90 in the same direction.

Convergence of the largest Lyapunov exponent is shown on Figure 6. Figure 7 represents Higuchi fractal dimension of observed signal.

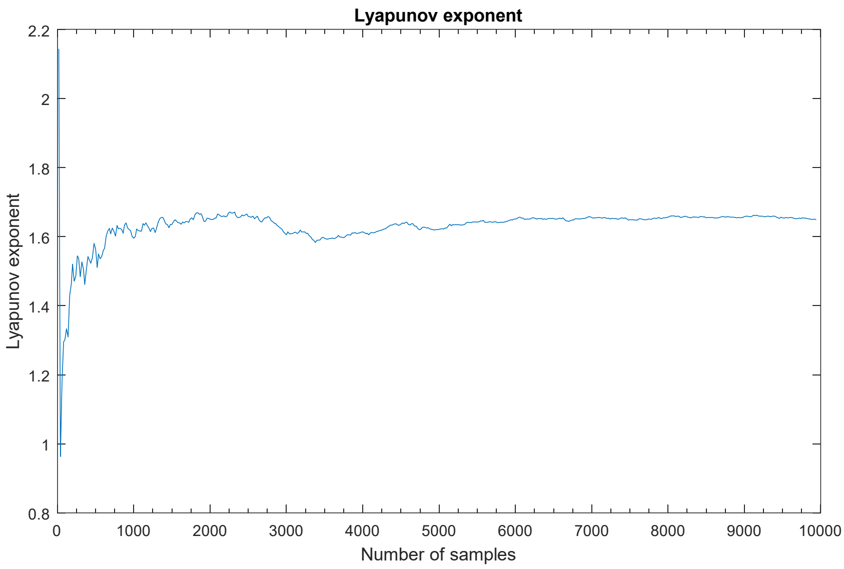

Figure 6. Convergence of largest Lyapunov exponent

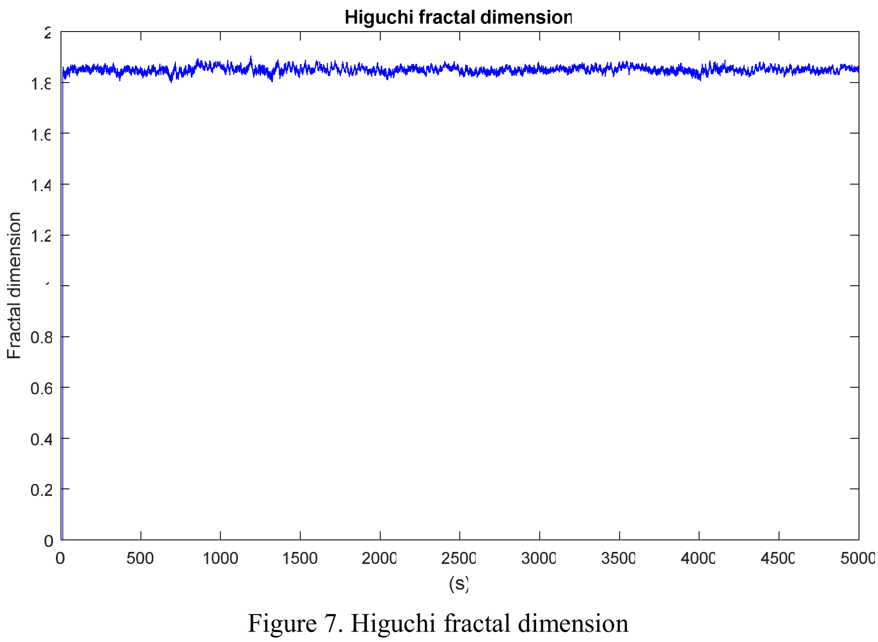

Figure 7. Higuchi fractal dimension

The results shown in Figures from 2 to 7 indicate the existence of chaos in the observed vibration signal.

GRATITUDE

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5. CONCLUSION

In this paper, the one set of data obtained by measuring the vibration of the rail under the seat of Gazela helicopter pilots was analyzed. By using quantitative and qualitative indicators of chaos, the existence of chaotic dynamics in the observed signal was confirmed.

Vibrations are the main causes of structural damage, electronics system failure and occupational diseases of pilots and permanent helicopter crews. Determining the nature of vibrations in a helicopter is of great importance in defining the different controllers used to control vibration. Also, determining the exact nature of vibrations in a helicopter, is important for the development of helicopter flight simulator. In many simulations until now it was thought that the vibrations have periodic nature.

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