A NOVEL METHOD FOR OPTIMIZATION OF PID/PIDC CONTROLLER UNDER CONSTRAINTS ON PHASE MARGIN AND SENSITIVITY TO MEASUREMENT NOISE BASED ON NON-SYMMETRICAL OPTIMUM METHOD

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Abstract: This paper presents a novel method for frequency domain optimization of PID controllers with a series lead-lag filter (PIDC). Optimization procedure is based on maximization of integral gain k_i under constraints to sensitivity to measurement noise M_n. The proposed method is based on the non-symmetrical optimum method (NSO) and provides a high degree of non-symmetrical optimum for the given phase margin ϕ_{pfz}. Solution to optimization procedure gives parameters of PIDC controller which give the minimum of IAE (Integrated Absolute Error). Efficiency of the proposed method is analyzed on large class of industrial processes.

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1. INTRODUCTION

The great importance and use of PI/PID controllers with participation of more than 94% in implementation of feedback loops in industry [1] lead to development of a large number of different methods for tuning their parameters. There have been developed efficient and simple procedures for tuning parameters of industry controllers, as well as optimization procedures [2-16] of the controllers with aim to minimize IAE (*Integrated Absolute Error*) under constraints to robustness, which satisfies criterion given in [17].

One of the well-known methods for designing PI/PID controllers applies the principle of non-symmetrical optimum (NSO) [18]. NSO principle is based on the requirement that phase Bode characteristics $\phi(\omega)$, ie. characteristics of the feedback function $\phi_{pf}(\omega)=180°+\phi(\omega)$ should be non-symmetrical in relation to the straight line drawn through the intersection point of gain (ω_1,0 dB), which is perpendicular to the frequency axis. Based on these facts it can be easily formed non-symmetrical criterion which implies that certain number of even derivatives of phase characteristics tend towards to zero in gain crossover frequency as it was pointed out in [19].

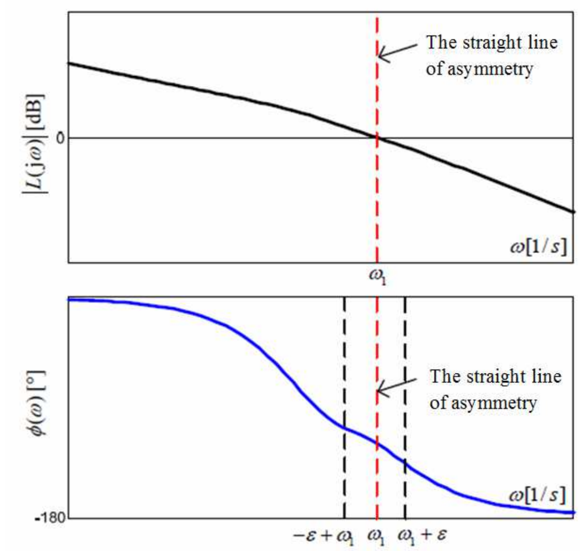


Fig. 1. Bode plots of feedback function $L(j\omega)$ illustrating NSO principle

Performance and robustness indices of control loops with PI/PID controllers can be further improved using PIDC controller [20,21]. Transfer function of PIDC controller is defined by an expression (1)

$$C_{\mathrm{PIDC}}(s) = (k + \frac{k_{\mathrm{i}}}{s} + k_{\mathrm{d}}s + k_{\mathrm{h}}s^2)F_{\mathrm{NF}}(s) \quad (1)$$

where k, k_i, k_d, k_h are proportional, integral, derivative gain of controller, respectively, and $F_{NF}(s)$ is low-pass filter.

This paper presents a novel design method of PIDC controller for certain industrial processes with and without transport delay. Presented method is based on the NSO principle under constraints to the phase margin and the sensitivity to the measurement noise M_n. The optimization procedure is aimed to realize a greater degree of asymmetry of the function $\phi_{pf}(\omega)$ around crossover frequency ω_1 such as indicated previously. The initial requirement is to perform a minimization of IAE with adequate robustness, and for that purpose max(k_i) method is applied. Parameters of the PIDC controller are determined on the basis of the specified phase margin ϕ_{pfz} and non-symmetrical criterion requirements

The proposed design method of PID/PIDC controllers is analyzed via numerical simulations of the certain class of static and astatic industrial processes with and without transport delay.

2. A NOVEL METHOD FOR OPTIMIZATION OF PIDC CONTROLLER BASED ON NSO PRINCIPLE

The control system structure with PIDC controller is presented in Fig. 2 for certain class of transfer functions of industrial processes. Transfer function $G_p(s)$ represents the process, r-reference signal, u-control signal, d-disturbance, n-measurement noise, y-output signal and $G_{ff}(s)$ describes feed forward from reference signal r to control signal u.

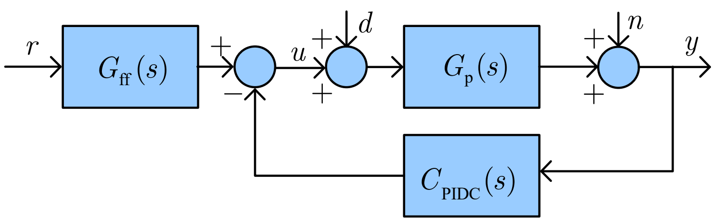


Fig. 2. Control structure with controller $C_{PIDC}(s)$

Feedback transfer function of the system from Fig. 1 is $L(s)=C_{PIDC}(s)G_p(s)$ which can further be written in the form

$$L(s) = \gamma \frac{k_h s^3 + k_d s^2 + ks + k_i}{sF_{NF}(s)} G_p(s) \tag{2}$$

where k, k_i, k_d, k_h are tunable parameters. In this paper, it is used low-pass filter of the second order with time constant T_f and relative damping factor $\zeta = 1/\sqrt{2}$, forms of

$$F_{\mathrm{NF}}(s) = \frac{1}{\frac{T_{\mathrm{f}}^2}{2}s^2 + T_{\mathrm{f}}s + 1} \tag{3}$$

If the static gain of the process $G_{\mathrm{p}}(s)$ is positive then parameter γ=1, while for negative static gain applies γ=-1. Without loss of generality, the proposed method considers the case γ=1.

Requirements to obtain desired performance/robustness of the closed-loop system can be presented as follows:

1. Phase margin $\phi_{\mathrm{pf}}=\phi_{\mathrm{pfz}}$,

$$\phi_{\mathrm{pf}}(\omega) = 180° + \arg L(\mathrm{j}\omega), \left|L(\mathrm{j}\omega)\right| = 1, \tag{4}$$

2. Time constant of filtration T_{f},

$$T_{\mathrm{f}} = \sqrt{\frac{2k_{\mathrm{h}}}{M_{\mathrm{n}}}}, \tag{5}$$

where M_{n} is sensitivity to measurement noise at high frequencies defined as

$$M_{\mathrm{n},\infty} = \lim_{\omega\to\infty}\left|\frac{C(\mathrm{j}\omega)}{1+C(\mathrm{j}\omega)G_{\mathrm{p}}(\mathrm{j}\omega)}\right| = \frac{2\left|k_{\mathrm{h}}\right|}{T_{\mathrm{f}}^2} \tag{6}$$

3. Non-symmetrical criterion in ideal case for function $\phi_{\mathrm{pf}}(\omega)$ can be expressed in general form as follows

$$\mu_{\mathrm{n}} = \left.\frac{\partial^{\mathrm{n}}\phi_{\mathrm{pf}}(\omega)}{\partial\omega^{\mathrm{n}}}\right|_{\omega=\omega_1} = 0,\ n = 2, 4, 6, ... \tag{7}$$

Taking into account that function ϕ_{pf} should have a great degree of assymetry (NSO principle) around crossover frequency ω_1, the prevoius criterion (7) can be eased. Hence, an optimization procedure of PIDC controller under constraints can be represented in arranged form (8)

$$\begin{aligned} &\max_{\omega,k,k_{\mathrm{i}},k_{\mathrm{d}},k_{\mathrm{h}}} (k_{\mathrm{i}}),\\ &\left|L(\mathrm{j}\omega)\right| = 1,\\ &180° + \arg L(\mathrm{j}\omega) = \phi_{\mathrm{pfz}},\\ &\mu_2(\omega,k,k_{\mathrm{i}},k_{\mathrm{d}},k_{\mathrm{h}}) = 0,\\ &\mu_4(\omega,k,k_{\mathrm{i}},k_{\mathrm{d}},k_{\mathrm{h}}) = 0. \end{aligned} \tag{8}$$

for specified phase margin ϕ_{pfz} and sensitivity to the measurement noise M_{n}. By introducing empirically dtermined initial values $\omega^*, k^*, k_{\mathrm{i}}^*, k_{\mathrm{d}}^*, k_{\mathrm{h}}^*$ in optimization procedure (8) with

(2), (3) and (6) parameters of the PIDC controller k, k_i, k_d, k_h and T_f are obtained, as well as the crossover frequency ω_1.

In similar way, optimization procedure of PID controller can be performed to determine parameters k, k_i, k_d and T_f. This design procedure of PID controller based on the principle on non-symmetrical optimum is elaborated in detail in [19] and can be expressed as follows

$$\begin{aligned} &\min_{\omega,k,k_i,k_d} \mu_4^2(\omega,k,k_i,k_d,) \\ &|L(j\omega)| = 1, \\ &180° + \arg L(j\omega) = \phi_{pfz}, \\ &\mu_2(\omega,k,k_i,k_d) = 0. \end{aligned} \tag{9}$$

3. SIMULATION ANALYSIS

The effectiveness of the presented PID/PIDC design procedure is verified via numerical simulations on eight processes $G_{p1}(s)$-$G_{p8}(s)$ including static and astatic processes with and without transport delay.

$$G_{p1}(s) = \frac{1}{(s+1)^4}, \quad G_{p2}(s) = \frac{1}{\prod_{k=0}^{3}(0{,}7^k s+1)}, \quad G_{p3}(s) = \frac{e^{-5s}}{(s+1)^3}, \quad G_{p4}(s) = \frac{1-s}{(s+1)^3},$$

$$G_{p5}(s) = \frac{9}{(s+1)(s^2+2s+9)}, \quad G_{p6}(s) = \frac{1}{\cosh\sqrt{2s}}, \quad G_{p7}(s) = \frac{1}{s(s+1)^3}, \quad G_{p8}(s) = e^{-\sqrt{s}}.$$

In order to get better response to a reference signal, the control structure from Fig. 1 can be adapted to have the following control signal $U(s)=k(bR(s)-F_{NF}(s))+k_i(R(s)-F_{NF}(s))/s-k_d sF_{NF}(s)$, where b is feedforward control parameter $0\leq b\leq 1$.

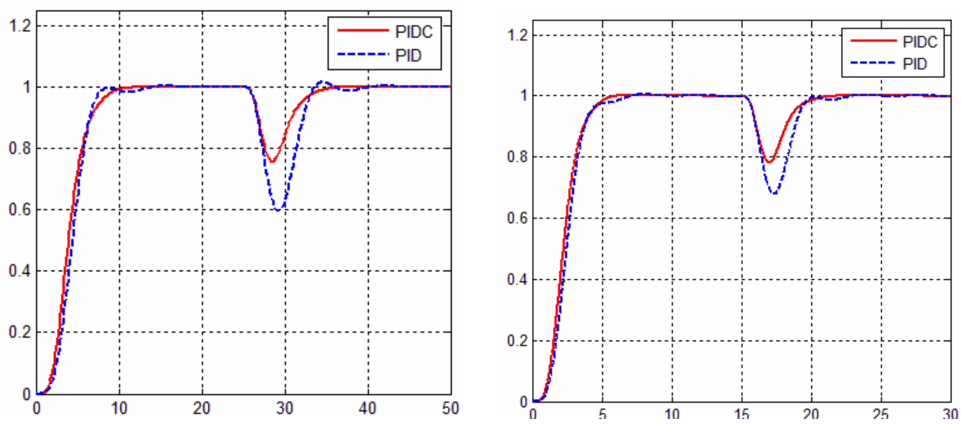
Performance/robustness of the closed loop system with PIDC controller is compared with those with PID controller, which parameters are also determined applying non-symmetrical criterion and optimization procedure (9) from [19].

Table 1. gives values of parameters of PID/PIDC controller for every process under constraints on phase margin and measurement noise, as well as maximum of the sensitivity function $M_s = \max_{\omega} |1/(1+L(j\omega))|$ and maximum of the complementary sensitivity function $M_p = \max_{\omega} |L(j\omega)/(1+L(j\omega))|$.

Table I Parameters of PIDC and PID controller obtained by the proposed method for $G_{pj}(s)$, j=1,2,…,8 where $T_f = \sqrt{2k_h / M_n}$ for PIDC and $T_f = k_d / M_n$ for PID

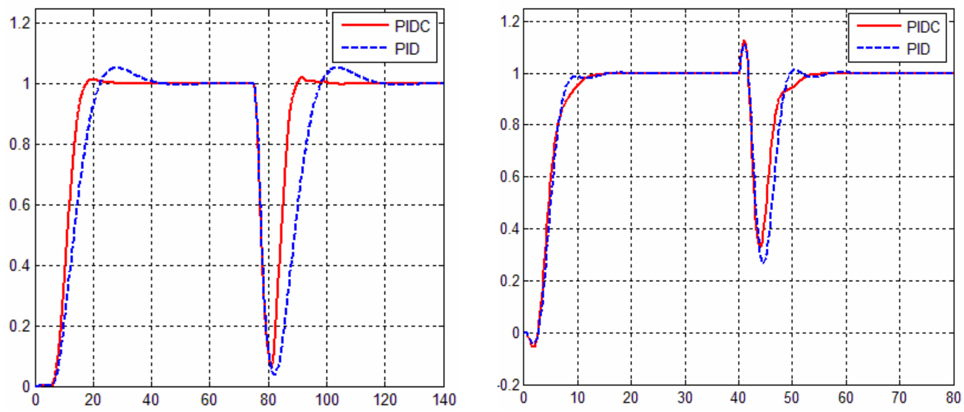
Process	k	k_i	k_d	k_h	ω_1	ϕ_{pfz}	M_n	M_s	M_p
$G_{p1}(s)$	3.7209	1.0467	4.6232	2.0782	0.9852	50	55	1.79	1.21
	1.7733	0.6056	1.3894	-	0.5806	50	25	1.79	1.21
$G_{p2}(s)$	4.3434	2.1001	3.1314	0.7406	1.7065	45	50	1.88	1.33
	2.5120	1.3395	1.3691	-	1.2008	45	25	1.94	1.36
$G_{p3}(s)$	0.7656	0.1520	1.4970	1.3122	0.1569	60	20	2.02	1.02
	0.2229	0.0932	0.1386	-	0.0931	60	11	1.63	1.00
$G_{p4}(s)$	1.3279	0.4168	1.2782	0.3116	0.5086	55	6	2.38	1.48
	0.9230	0.3488	0.5709	-	0.3845	55	3	1.99	1.19
$G_{p5}(s)$	2.2824	2.2543	0.6566	0.2006	1.8779	60	8	1.49	1.05
	1.0800	1.7079	0.2317	-	1.3536	60	4	1.91	1.07
$G_{p6}(s)$	9.5144	14.6097	1.0748	0.0639	8.1443	45	50	2.40	1.44
	6.5954	8.6512	0.4973	-	6.8538	40	25	2.23	1.57
$G_{p7}(s)$	1.2025	0.1265	1.8298	1.4419	0.6732	40	40	1.58	1.54
	0.8026	0.0547	1.2536	-	0.6412	30	12	2.59	2.05
$G_{p8}(s)$	13.5378	40.4082	0.5716	0.0037	13.7335	40	40	2.18	1.54
	10.1401	30.9532	0.2860	-	10.6275	40	20	2.11	1.51

Fig. 2-5 show comparison of step responses to a reference signal and disturbance of the closed-loop system with PID and PIDC controller.



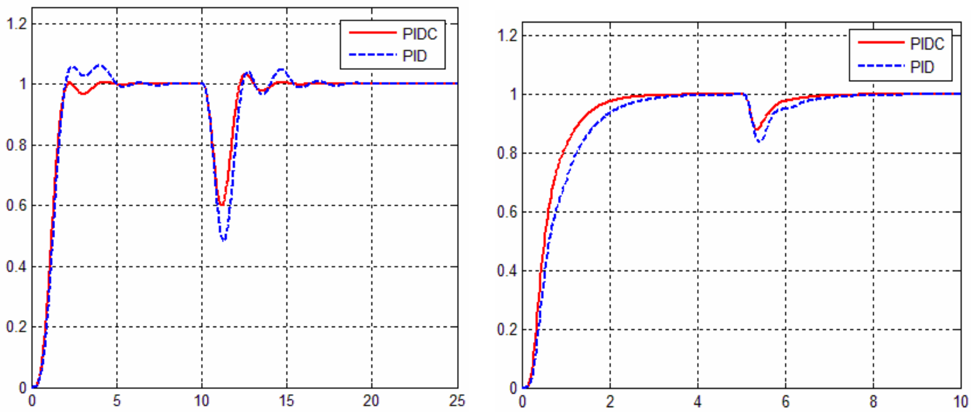
a) *b*)

Fig. 3. Comparison of step responses to a reference signal $r(t)$=1 and disturbance $d(t)$ with PIDC controller (red thick line) and PID controller (blue dashed line); *a*) $d(t)$=1 (t>25 s) for process $G_{p1}(s)$ and b=0; *b*) $d(t)$=1 (t>15 s) for process $G_{p2}(s)$ and b=0



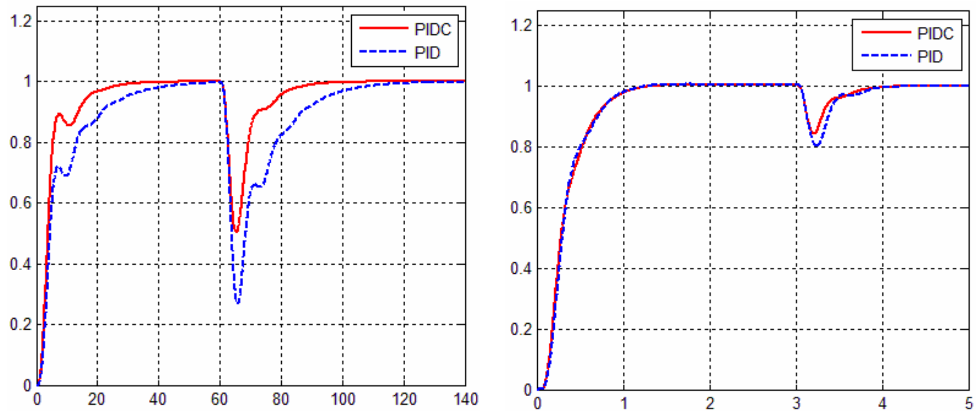
a) *b*)

Fig. 4. Comparison of step responses to a reference signal $r(t)$=1 and disturbance $d(t)$ with PIDC controller (red thick line) and PID controller (blue dashed line); *a*) $d(t)$=1 (t>70 s) for process $G_{p3}(s)$ and b=0; *b*) $d(t)$=1 (t>40 s) for process $G_{p4}(s)$ and b=0



a) *b*)

Fig. 5. Comparison of step responses to a reference signal $r(t)$=1 and disturbance $d(t)$ with PIDC controller (red thick line) and PID controller (blue dashed line); *a*) $d(t)$=1 (t>10 s) for process $G_{p5}(s)$ and b=0; *b*) $d(t)$=1 (t>5 s) for process $G_{p6}(s)$ and b=0



a) *b*)

Fig. 6. Comparison of step responses to a reference signal $r(t)$=1 and disturbance $d(t)$ with PIDC controller (red thick line) and PID controller (blue dashed line); *a*) $d(t)$=0.5 (t>60 s) for process $G_{p7}(s)$ and b=0.4; *b*) $d(t)$=2 (t>3 s) for process $G_{p8}(s)$ and b=0

4. CONCLUSIONS

Proposed design method for optimization of PIDC controller is based on the principle of the non-symmetrical optimum and max(k_i) method. By applying this procedure, adequate performance and robustness indices of the closed loop system are achieved for static and astatic industrial processes with and without transport delay. Obtained results of numerical simulations show effectiveness of the presented design procedure for all stable processes except those which are integral. It is also shown a superiority of PIDC controller over PID controller regarding obtained performance/robustness indices which is obviously according to Figs. 3-6. It should be noted that this design procedure is comparable with optimal tuning methods [20-21], but also with other optimization methods [2-16] on the large class of industrial processes.

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