A NEW ALGORITHM FOR ADAPTIVE BEAMFORMING

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Abstract: A new beamforming algorithm that is consisted of the adaptive vector and single adaptive complex coefficient is proposed. By using the adaptive vector, the interfering signals are cancelled. At the same time, aimed to adjust the gain in the direction of the desired signal, the complex coefficient is updated. Simulation results show that the proposed algorithm exhibits faster convergence speed and smaller steady state error compared to the LMS beamformer.

1. INTRODUCTION

Smart antenna systems consist of antenna array with signal processing algorithm used to automatically optimize and adjust radiation patterns to the concrete signal scenario, [1]. The successful adaptive antenna system design depends on the choice and performance of the beamforming algorithm. The aim of the adaptive algorithm is to direct the main beam of the radiation pattern in direction of the desired signal, while placing nulls in the directions of interferers, [2].

Till now many adaptive algorithms for synthesis of the antenna radiation patterns are developed. Due to its computational simplicity, the Least Mean Square (LMS) is one of the most used adaptive algorithms, [3, 4]. The main disadvantage of the LMS algorithm is slow convergence and therefore slow tracking capabilities of the moving interference sources, [5]. In [5] the Combined LMS-LMS beamforming algorithm is proposed, in order to increase convergence speed of the conventional LMS. In the Combined LMS-LMS

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algorithm, beside the reference signal, the steering vector of the desired signal should be known, [5].

In this paper the adaptive beamforming algorithm based on the modified block diagram of the conventional LMS beamformer, is proposed. The proposed algorithm consists of the adaptive vector and the adaptive complex coefficient that are updated in order to minimize the error at the beamformer output. Simulation results show that the proposed algorithm exhibits better performance compared to the LMS, in the sense of the convergence speed and steady state error.

The paper is organized as follows. In the Section 2 a brief introduction to the LMS beamforming algorithm is given. In Section 3 the proposed algorithm is presented, whereas in Section 4 and Section 5 the simulation results and conclusions are respectively given.

1. LMS BEAMFORMING

Consider a linear uniform antenna array with N equidistant elements with inter-element spacing d. The antenna receives one desired signal and M interference signals. The received signal can be written as:

$$\mathbf{x}(n) = \mathbf{a}(\theta_0)s(n) + \mathbf{A}(\boldsymbol{\theta}_i)\mathbf{s}_i(n) + \boldsymbol{\eta}(n), \tag{1}$$

where $\mathbf{x}(n)$ is the vector with dimensions $N\times 1$, that is consists of received symbols from the corresponding antenna elements, $s(n)$ is the desired signal, $\mathbf{s}_i(n)$ is the vector of the interfering signals from M sources and $\boldsymbol{\eta}(n)$ is the vector consisting of samples of a white Gaussian noise. The steering vector of the desired signal is denoted by $\mathbf{a}(\theta_0)$, with the elements that are a function of the incident angle of the desired signal:

$$\mathbf{a}(\theta_0) = \begin{bmatrix} 1 & e^{j\psi_0} & e^{j2\psi_0} & \cdots & e^{j(N-1)\psi_0} \end{bmatrix}^T, \tag{2}$$

where ψ_0 is the phase difference between two adjacent antenna elements:

$$\psi_0 = \frac{2\pi d}{\lambda}\sin\theta_0. \tag{3}$$

In the similar way, the steering matrix $\mathbf{A}(\boldsymbol{\theta}_i)$ is defined, where $\boldsymbol{\theta}_i$ is the vector of the incident angles of the interference signal.

The received signal $\mathbf{x}(n)$ is multiplied with the beamformer coefficients $\mathbf{w}$, giving the output signal $y(n)$:

$$y(n) = \mathbf{w}^H\mathbf{x}(n). \tag{4}$$

The beamformer coefficients should be adjusted in such a way that the output signal $y(n)$ is equal to the desired signal $s(n)$. In the other words, the interference signals should be cancelled. If the sequence of the desired signal is known (pilot signal $d(n)$), the beamformer coefficients can be obtained by minimizing the mean square error (MSE), [2]:

$$J(\mathbf{w}) = \mathrm{E}\{e(n)^2\} = \mathrm{E}\{|\, d(n) - \mathbf{w}^H\mathbf{x}(n)\,|^2\}. \tag{5}$$

The optimal solution which minimize (5) is equal to:

$$\mathbf{w}^* = \mathbf{R}^{-1}\mathbf{p}, \tag{6}$$

where **R** is the autocorrelation matrix of the input signal, and **p** is the cross-correlation vector between the input and desired signal. Optimal vector (6) can be found iteratively by using the LMS algorithm, [2]:

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \mu e(n)\mathbf{x}(n), \tag{7}$$

where μ is the algorithm step size.

The LMS convergence speed depends on the eigenvalue spread, that is defined as the ratio of the maximal and minimal eigenvalue of autocorrelation matrix **R**. The smaller eigenvalue spread, the faster convergence speed [6]. The autocorrelation matrix of the input signal is equal to the sum of three terms: autocorrelation matrix of the desired signal, autocorrelation matrix of the interference signals and autocorrelation matrix of the noise:

$$\mathbf{R} = \mathbf{R}_s + \mathbf{R}_i + \mathbf{R}_\eta. \tag{8}$$

The fastest convergence speed is achieved in the case when matrix (8) is diagonal, [6].

2. THE PROPOSED ALGORITHM

Block diagram of the proposed beamforming algorithm is shown in Fig 1. The adaptive vector $\mathbf{w}(n)$ is updated in the following way:

$$\mathbf{w}(n+1) = \mathbf{w}(n) - \mu y_m(n)\mathbf{x}_m(n), \tag{9}$$

where $\mathbf{x}_m(n)$ is the vector of the modified input signal:

$$\mathbf{x}_m(n) = \mathbf{x}(n) - \mathbf{a}(\theta_0)d(n), \tag{10}$$

whereas $y_m(n)$ is defined as:

$$y_m(n) = \mathbf{w}^H\mathbf{x}_m(n). \tag{11}$$

The steering vector of the desired signal is denoted by $\mathbf{a}(\theta_0)$ and it is assumed that is known. If the desired user is moving or the steering vector is unknown, then it should be iteratively estimated [5].

The vector **w** converges toward the solution that places radiation pattern nulls in direction of the interfering signals, while minimizing the noise at the beamformer output.

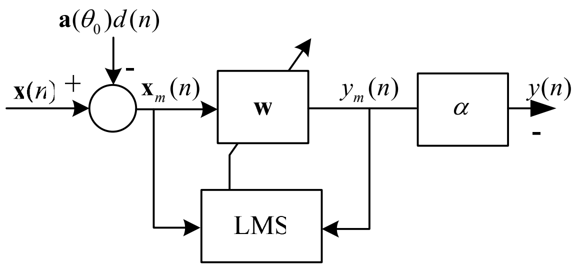


Figure 1. Block diagram of the proposed beamformer

The coefficient α is updated in the following way:

$$\alpha(n) = \frac{1}{\mathbf{w}(n)^H \mathbf{a}(\theta_0)}, \tag{12}$$

The coefficient α is used to normalize the radiation pattern in such a way than the gain in the desired signal direction is equal to one. In the other words, the coefficient α ensure that output signal, obtained when $\mathbf{x}$ is applied at the beamformer input, is equal to the desired signal:

$$e(n) = d(n) - \alpha \mathbf{w}^H \mathbf{x}(n) = 0. \tag{13}$$

From (13) it follows:

$$\alpha(\infty)\mathbf{w}(\infty) = \mathbf{R}^{-1}\mathbf{p} \ , \tag{14}$$

i.e. the beamformer $\alpha\mathbf{w}$ converges to the same solution as the LMS beamformer, in the mean sense.

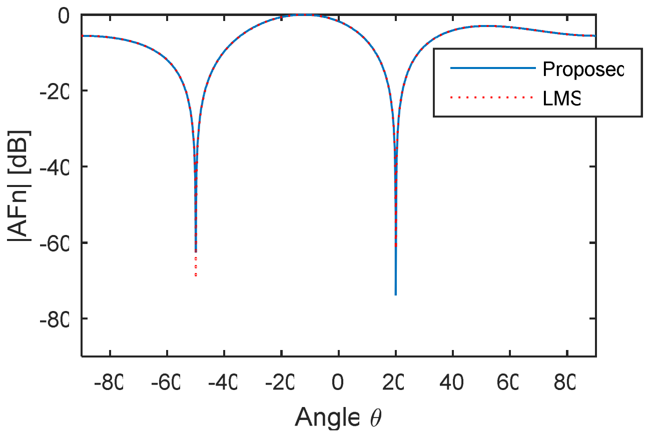
Unlike the conventional beamforming LMS algorithm, the convergence speed of the proposed algorithm depends on characteristics of matrix $\mathbf{R}_m$ that is equal to the sum of the autocorrelation matrix of the interference signals and autocorrelation matrix of the noise. In the other words, the influence of the autocorrelation matrix of the desired signal is annulled. In this way, the faster convergence speed is achieved. The maximal step size is equal to:

$$\mu_{\max} = \frac{1}{tr[\mathbf{R}_i + \mathbf{R}_\eta]}. \tag{15}$$

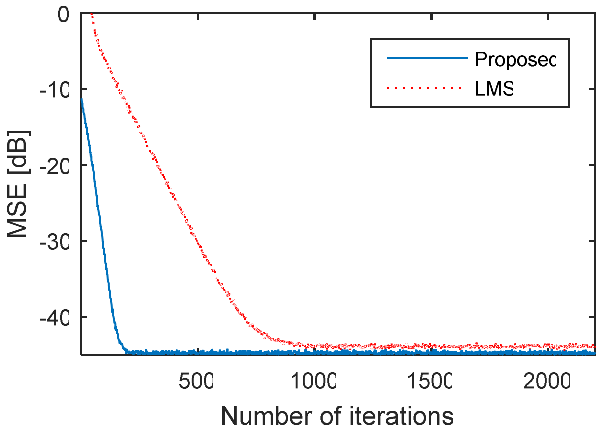
3. SIMULATION RESULTS

In all simulations, the desired signal given by (1) is used. The uniform linear antenna array with N=3, N=5 and N=10 antenna elements, separated by equal distance $d=\lambda/2$, is analyzed. The variance of white Gaussian noise is 0.0001. The radiation patterns obtained by LMS and proposed algorithm are given in the normalized values. The algorithm step size is set to μ=0.01.

Figs. 2.a and 2.b show radiation pattern and MSE of the considered algorithms for N=3, θ_0=45° and $\boldsymbol{\theta}_i$=[-50° 20°]. It can be seen that both of the algorithms have similar radiation patterns, where the pattern of the proposed one has a higher attenuation in the interference directions. For this reason, the proposed algorithm exhibits smaller MSE in the steady state (Fig. 2.b). The eigenvalues spread of the matrices $\mathbf{R}$ and $\mathbf{R}_m$ are 8.45 and 1.84, respectively. From Fig. 2.b it can be seen that the proposed algorithm has faster convergence that is in accordance with the theoretical expectations.



a) radiation pattern

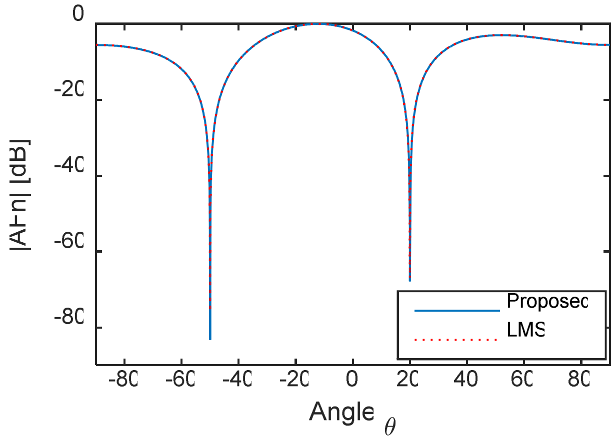


b) MSE

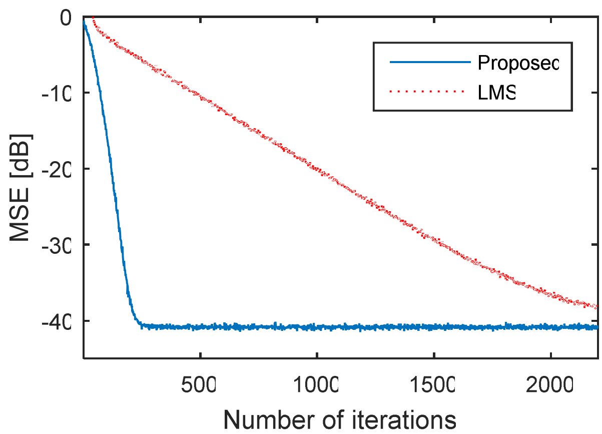
Fig 2. Comparison of the proposed algorithms for N=3

Simulation results for N=3, θ_0=30° and $\boldsymbol{\theta}_i$=[-50° 20°] are shown in Figs. 3.a and 3.b. The eigenvalue spread of matrices $\mathbf{R}$ and $\mathbf{R}_m$ are 32.31 and 1.84, respectively. It can be seen that convergence speed of the proposed algorithm is same as in the previous example, because the convergence speed does not depend on the direction of the desire signal. On the other way, the LMS algorithm converges slower, because the eigenvalue of $\mathbf{R}$ in this example in increased. From the standpoint of the steady state error, the same conclusion as in the previous example can be given here.

Simulation results for N=5, θ_0=-20° and $\boldsymbol{\theta}_i$=[-50° 10° 20° 60°] are shown in the Figs. 4.a and 4.b. The eigenvalue spread of matrices **R** and $\mathbf{R}_m$ are 7.67 and 7.34, respectively. In this example, both algorithms exhibit fast convergence. In the other way, the proposed algorithm has smaller steady state error.

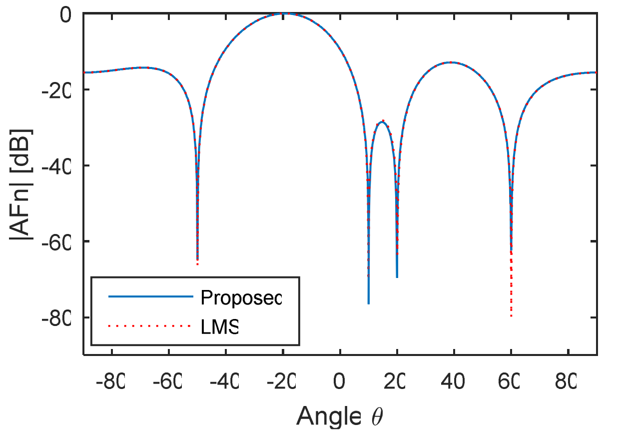


a) radiation pattern

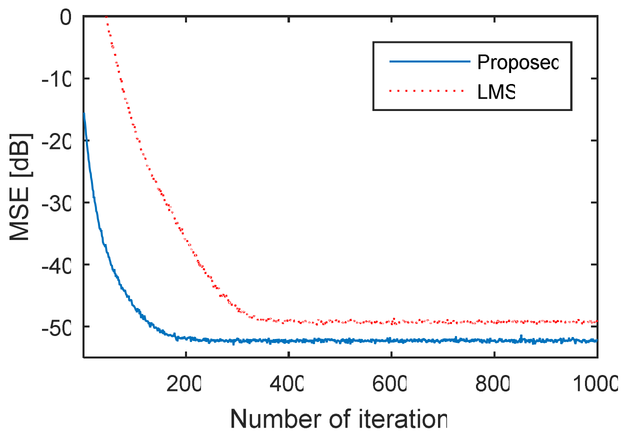


b) MSE

Fig 3. Comparison of the proposed algorithms for N=3

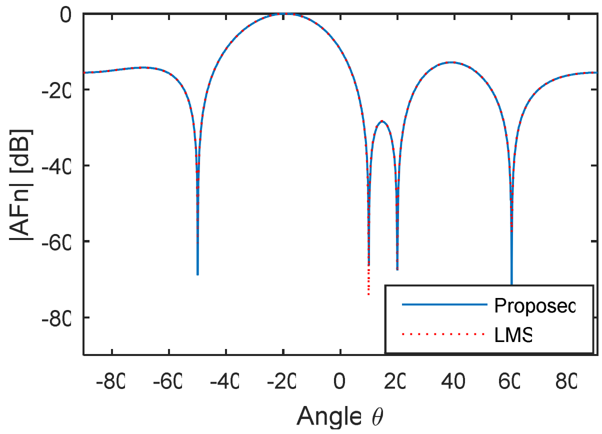


a) radiation pattern

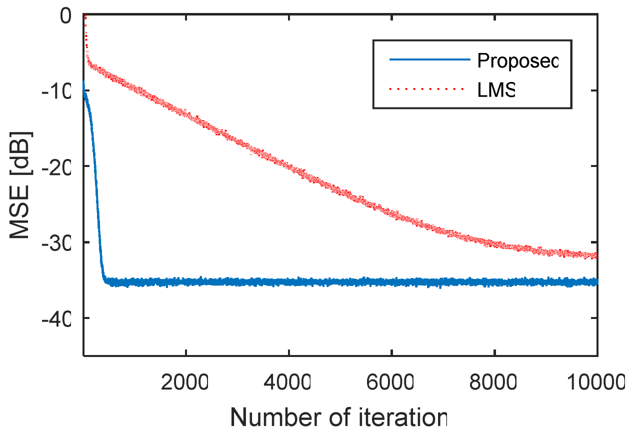


b) MSE

Fig 4. Comparison of the proposed algorithms for N=5

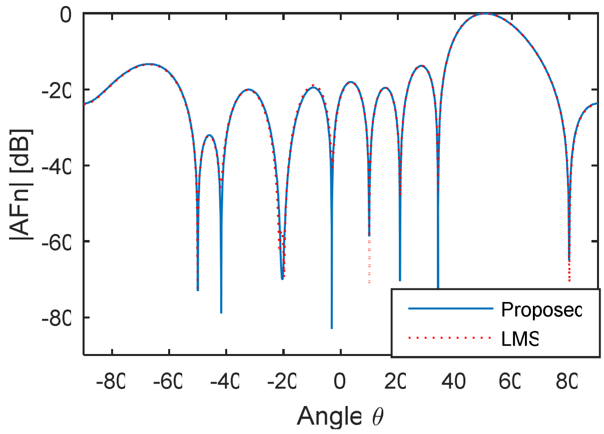


a) radiation pattern

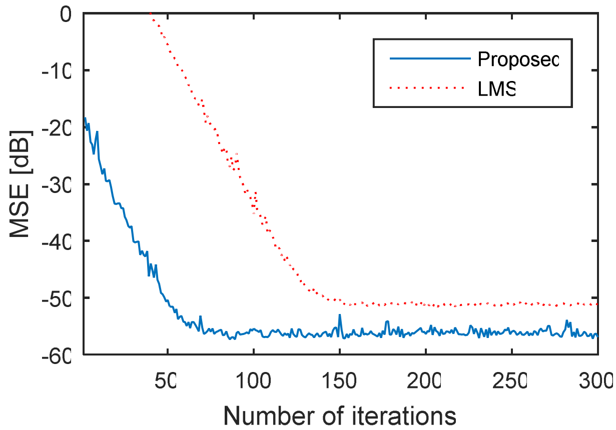


b) MSE

Fig 5. Comparison of the proposed algorithms for N=3



a) radiation pattern



b) MSE

Fig 6. Comparison of the proposed algorithms for N=10

Figs. 5.a and 5.b shows radiation patterns and MSE of the considered algorithms for N=5, θ_0=30° and $\boldsymbol{\theta}_i$=[-50° 10° 20° 60°]. In this example the eigenvalue spread of matrices **R** and $\mathbf{R}_m$ are 166.17 and 7.34, respectively. Since the eigenvalue spared of **R** is increased, the improvement of the proposed algorithm compared to the LMS is more noticeable.

Finally, Figs. 6.a and 6.b show radiation patterns and MSE for considered algorithms for N=10, θ_0=50° and $\boldsymbol{\theta}_i$=[-50° 10° 20° 80°]. It can be observed that the proposed algorithm has faster convergence and significantly smaller MSE, which corresponds to the smaller radiation pattern values in the interference directions.

4. CONCLUSION

The adaptive beamforming algorithm, based on the modified block diagram of the conventional LMS beamformer, is proposed. The proposed solution consists of the adaptive vector and adaptive complex coefficient. The adaptive vector is used to cancel the interference signals, whereas the adaptive complex coefficient is used to adjust the antenna gain in the desired signal direction.

Simulation results show that the convergence speed of the proposed algorithm does not depend on the desired signal direction, which is in accordance with the theoretical considerations. It is shown that, in different simulation scenarios, the proposed algorithm exhibits better performances compared to the LMS algorithm.

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