INHARMONICITY OF THE COPY OF ANTONIUS STRADIVARIUS VIOLIN

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Keywords: *Fundamental frequency, Harmonicity, Inharmonicity.*

Abstract: This paper presents the algorithm for estimation of inharmonicity of the string musical instruments. The second part of this paper presents the results of the application of this algorithm in estimation of inharmonicity of the copy of Antonius Stradivarious violin built in the transition period from the 19th to the 20th century. The analysis was made for two ways of string arousal: a) by using a bow and b) by using the piccicato technique. After that inharmonicityof the instrument was analysed in the third octave when played by using the pizzicato technique. The results are given in tables and graphics. In the end the comparative analysis of results was made.

1. INTRODUCTION

The string instruments produce a tone, i.e. they generate an acoustic wave by oscilating the string. Oscilation of a string is well explained in physics and acoustics, substantiated by a corresponding mechanical and acoustical model and described by adequate equations. After the mechanical arousal (the string is put out of balance (or set into vibration) by picking, striking or by a fiddle-bow) the string begins to oscillate tending to subside and come back into the equilibrium. The string is vibrating with the basic or fundamental frequency that depends on the dimensions of the string (length and diameter), the material it is made of and the tensile strength. Along with the fundamental frequency, because of the cimplex oscilating of the string (appearing of waves on 1/2, 1/4, 1/8, ...of the string length) there are acoustic components of the waves on the frequencies that represent the integer multipliers of the fundamental frequency. In the techniques these spectral components are called charmonics. The theory of music describes the complex structure of the string oscilating and the generated tone by means of aliquotes (**lat.** *aliquoties* - several times), where the aliquotes represent charmonic components of the generated tone. The detailed mathematical analysis shows that within one tone there are thanks to the aliquotes all tones

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included (the aliwuotes of various tomes overlap mutually). Different numbers of the present aliquotes and their different relative intensity within the total sounding determine the timbre of the sound [1]. Aliquotes are also called partial tones, i.e. the partials.

The detailed analysis of the oscilation of the string shows that because of the parameters of the string and the tensile strength the frequencies of the harmonics are not with the integer multiplier, i.e. there is inharmonicity. Inharmonicity of the oscilation of the string can be described by the coefficient of inharmonicity. Appearing of inharmonicity inevitably leads to the disturbing of aliquocy. Since there is inharmonicity in some instruments, the partials can be with frequencies that are: a) integer multipliers of the fundamental frequency (harmonics) and b) non-integer multipliers (inharmonics) [2]. A tone produced on an inharmonic instrument is not necessarily unpleasant. In [3] there is a statement that *softly* inharmonic tone possesses *certain warmth*

The quality of the string musical instruments can be, in addition to other parameters, expressed by means of the degree of inharmonicity, too. The strings are trightened by the great force so that their elasticity is decreased. The result of this is that the frequecy positions of the partials are on the positions of non-integer multipliers of the fundamental frequency. Consequently, the instrument with such strings is not harmonical. Along with the rigidity of the string, another factor for increasing inharmonicity is the character of the acoustic impedance of the resonating plate of the piano or the resonating body of the guitar, violine etc [3]. Inharmonics are outstanding at tones with lower frequencies. On higher frequencies it is harder to detect inharmonicity. The reason for a good detecting at tones with lower frequency lies in the fact that a great number of partials is within the range in which man can hear well. At tones with higher frequencies only a few first partials which do not express great inharmonicity are found in this region. On the base of the analysis of inharmonicity it is possible to make conclusions about the kind of the instrument that produces a tone. In [4] an algorithm for the automatic classigication of tones of the pioano and the guitar is described. The guitar as well as some other instruments may produce the same tone on three different strings. By the analysis of the degree of inharmonicity it is possible to determine the played tone, the string and the field where the string was pressed [5].

This paper presents the results of the analysis of inharmonicity of the violin, actually a copy of the Antonius Stradivarius violin, built in the Czeck Republik in the transition period from the 19th to the 20th century. The first part of the paper deals with the algorithm proposed by the authors of the paper [6]. That algorithm consists of two entities: a) estimation of the frequency position of partials and b) estimation of inharmonicity. Estimation of the frequency position is based on the peaks of the maximum within the spectrum and on the application of PCC of the convulsive kernel. Parameters of this part of the algorithm represent a part of the results found by the authors and published in [7] and [8]. Out of a great number of published formulas for calculating the inharmonicity the authors have applied the formula from [9]. After that the algorithm for estimation of the inharmonicity of the double stop published in [12] was described. The second part of the paper deals with the analysis of inharmonicity of the violin when the strings are aroused to vibrate by: a) afiddle-bow and b) by picking (pizzicato technique). Then the analysis of inharmonicity of the double stops from the third octave played on the first and third strings by the pizzicato technique was done. The obtained results are compared to the results of inharmonicity of the concert piano Steinway D [10], pianino August Förster (built in the Czeck Republic in 1971) [11] and the electric guitar Fender Stratokaster [6].

The organisation of the work is as follows. In the section 2 the algorithm for the estimation of inharmonicity is presented. In the section 3 the experimental results and the comparative analysis are presented. The section 4 represents a conclusion.

2. INHARMONICITY AT THE STRING MUSICAL INSTRUMENTS

A. The inharmonicity coefficient of the vibrating string

The theory of music implies harmonicity in defining the frequency composition of a tone, i.e. that the harmonics (partials) are the integer multipliers of the fundamental frequency, which mathematically can be presented as:

$$f_k = k \cdot f_0,\ \ \mathrm{k} = 1,2,..., \tag{1}$$

where f_0 is the fundamental frequency, k. the ordinal number of a partial and f_k the frequency of the partial. The frequency shifting of the partial from the frequency position of the harmonic represents the inharmonicity of a tone. Inharmonicity is defined by the coefficient of inharmonicity β:

$$f_k = k \cdot f_0 \sqrt{1 + \beta \cdot k^2},\ \ \mathrm{k} = 1,2,... \tag{2}$$

The coefficient of inharmonicity β depends on the material the string is made of and can be calculated on the base of:

$$\beta = \frac{\pi^3 \cdot Q \cdot d^4}{64 \cdot l^2 \cdot F}, \tag{3}$$

where Q is Jung's modul of elasticity of the material the string was made of, d the diameter of the string, l the length of the string and F the tension force.

B. The algorithm for estimation of the partials

The algorithm for estimation of partials is based on the spectrum analysis of the signal **x**. First of all the the spectrum **X** is calculated by using of the discrete Fourier transformation. After that the position of the maximal spectral component that represents the fundamental frequency is determined by the method of spectrum peaking [7]. In order to increase the estimation precision the parametric cubic convolution is applied and the fundamental frequency $\boldsymbol{f_0}$ is calculated [7, 8]. Estimation of the partials is performed by the algorithm shown in Fig.1. That algorithm consists of the following steps:

Input: Spectrum $\boldsymbol{X_\mathrm{b}}$ of the frame $\mathbf{x_b}$ of the signal **x**, the estimated fundamental frequency f_0, number of partials N_P.

Outout: partials f_P.

FOR p=1:N_P
Step 1: peaking of the p-th spectral component
Step 2: Counting of the p-th partial by using PCC interpolation.
END FOR p

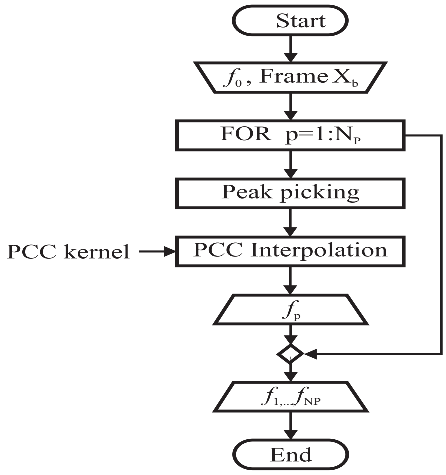


Fig. 1. Algorithm for estimation of the partials.

C. Algorithm for estimation of inharmonicity

The algorithm for estimation of inharmonicity consists of the following steps [6,9]:

Input: The fundamental frequency $\mathbf{f_0}$, partials f_P, number of partials N_P.
Output: The coefficient of inharmonicity β.
Step 1: Determination of frequencies of the harmonic components:

$$f_k = k \cdot f_0, \quad \mathrm{k} = 1,2,...,\mathrm{N_P}\,. \tag{4}$$

Step 2: Counting of the coefficient of inharmonicity according to the algorithm from [9]:

$$\beta = \frac{\left(f_k \dfrac{m}{k}\right)^2 - f_m^2}{k^2 f_m^2 - m^2 \left(f_k \dfrac{m}{k}\right)^2}\,, \tag{5}$$

where m and k are partials and . f_m and f_k the corresponding frequencies of partials.

D. Algorithm of estimation of two-tone aliquote distortion

Two-tones represent simultaneous sounding of two tones. Theory of aliquotes [1] sais that spectral content of one tone consists of harmonics which are, in the same time, harmonics of other tones. Simultaneous sounding of more tones means spectral overlapping of corresponding aliquotes. But, due to existence of some tones inharmonicity, aliquotes get untuned. Discrepancy of one tone's aliquotes related to other tone's corresponding aliquotes, inevitably leads to distortion of reproduced two-tone.

Algorithm for calculation of aliquote two-sound distortion consists of following steps [12]:

Input: aliquotes of two tones $f_{1,k}$ and $f_{2,k}$ where k=1:N_p, and N_p is the number of aliquotes analyzed.

Output: mean aliquote frequency error (MAFE), mean aliquote cent error (MACE), instrument's inharmonicity $\overline{\beta}_P$.

Step 1: detection of tones which form two-tone,

$$f_{1,1} \Rightarrow ton\_1;\; f_{2,1} \Rightarrow ton\_2\,, \tag{6}$$

Step 2: Calculation of mutual aliquotes:

$$\begin{aligned} & f_{1,k} = k \cdot f_{1,1};\; f_{2,l} = l \cdot f_{2,1} \\ & \Rightarrow k \cdot f_{1,1} = l \cdot f_{2,1} \\ & \Rightarrow \{(k,l)_g, k = 1,\dots,N_p, l = 1,\dots,N_p, g = 1,\dots,G\} \end{aligned}\,, \tag{7}$$

where G is number of pairs (k,l) which fulfill condition of equality of aliquotes.

Step 3: Mean aliquote frequency error:

$$MAFE = \frac{1}{G}\sum_{g=1}^{G} \left| f_{1,k} - f_{2,l} \right|_g\,. \tag{8}$$

Mean aliquote cent error:

$$MACE = \frac{1}{G}\sum_{g=1}^{G} \left| ftocent(f_{1,k}, f_{1,1}) - ftocent(f_{2,l}, f_{1,1}) \right|_g\,, \tag{9}$$

where *ftocent* is transformation function of frequency axis into cent axis with normalization to $f_{1,1}$ frequency.

Step 4: Inharmonicity calculation of all two-tones in contra octave:

$$\overline{MAFE} = \frac{1}{N_D} \sum_{d=1\_1}^{N_D} MAFE_d \ , \tag{10}$$

$$\overline{MACE} = \frac{1}{N_D} \sum_{d=1\_1}^{N_D} MACE_d \ , \tag{11}$$

where N_D is a number of two-tones analyzed.

Step 5: Calculation of instrument's inharmonicity as a mean value of some tone's inharmonicity coefficients:

$$\overline{\beta}_P = \frac{1}{N_S} \sum_{S=ton\_1}^{ton\_N_S} \beta_S \ , \tag{12}$$

where S is a continuum of tones analyzed, N_S number of continuum S elements, and β_S inharmonicity factor of corresponding tone.

3. EXPERIMENTAL RESULTS AND ANALYSIS

Further in the paper the coefficients of inharmonicity of the copy of Antonius Stradivarius violin, built in the Czeck Republick (according to the expertise performed in Germany in 1993) (Fig. 2). The strings set on it are of the type Dominant Set 135 B, the worldwide famous producer Thomastic Infeld Vienna (the string $\boldsymbol{e}^2$: chrome steel, ball end, the string $\boldsymbol{a}^1$: synthetic core, aluminium wound, the string $\boldsymbol{d}^1$: synthetic core, silver wound, the string ***g***: synthetic core, silver wound). The analysis will imply the estimation of inharmonicity of all strings and determination of the mean value that will represent inharmonicity of the instrument for two ways of string arousing: a) playing with the fiddle-bow and b) playing by the pizzicato technique (by picking the string). In addition to that the analysis of iharmonicity for the double stops from the third octave with the pizzicato technique will be done. The double stops were performed by the simultaneous playing of tones on the first and second strings.

For the purpose of the experiment a base of signals was formed. On the signals the algorithms described in section 2 was applied. The parameters of the algorithm are T=0.66s, NFFT=10*218, m=6, k=10.

Fig. 2. The tested violin.

A. Base

The base consists of all the tones (one octave) on the strings E5 (e^2), A4 (a^1), D4 (d^1) and G3 (g). The recording was done with the measuring frequency f_s=44.1 kHz and 16 bits/sample.

B. Results

In Fig. 3 and Fig.4 there are diagrams related to the tones obtained by playing with the fiddle-bow (Fig. 3 (G3), Fig. 4 (D4)) while in Fig. 5 and Fig. 6 there are time diagrams of the tones obtained by the pizzicato technique (Fig. 5 (G3), Fig. 6 (D4)). In every figure a) represents the time form of the complete signal, b) the time form of the frame lasting 32 ms, c) the difference between the frequencies of harmonic and inharmonic components and d) the part of amplitude characteristic to the 10^{th} harmonic. The vertical line denotes the position of the harmonic component. In Tab. I there are values of inharmonicity calculated by using of the algorithm described in sectionb 2. Fig. 7 shows frequency positions of harmonics and inharmonics aliquotes for playing E5-G#5, while Fig 8 shows positions of notes. Positions of inharmonic aliquotes are presented at fig. 9.a (aliquote G#7), fig. 9.b (aliquote G#8), fig. 9.c (aliquote G#9). Values of semitones inharmonicity coefficients are displayed in table II. Frequency and cent values of differences of harmonious and inharmonious aliquotas are given in table III.

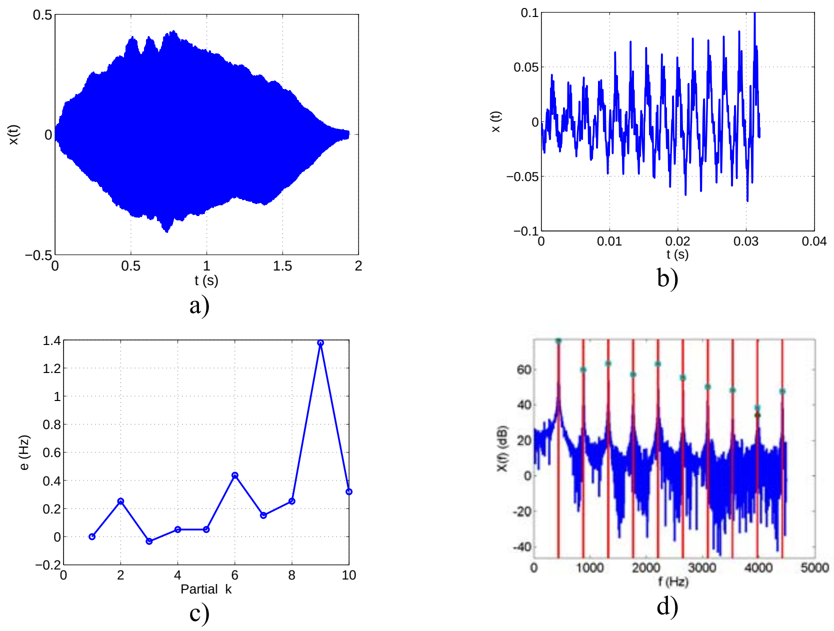


Fig. 3. Signal of the tone G3 produced by playing with the fiddle-bow.

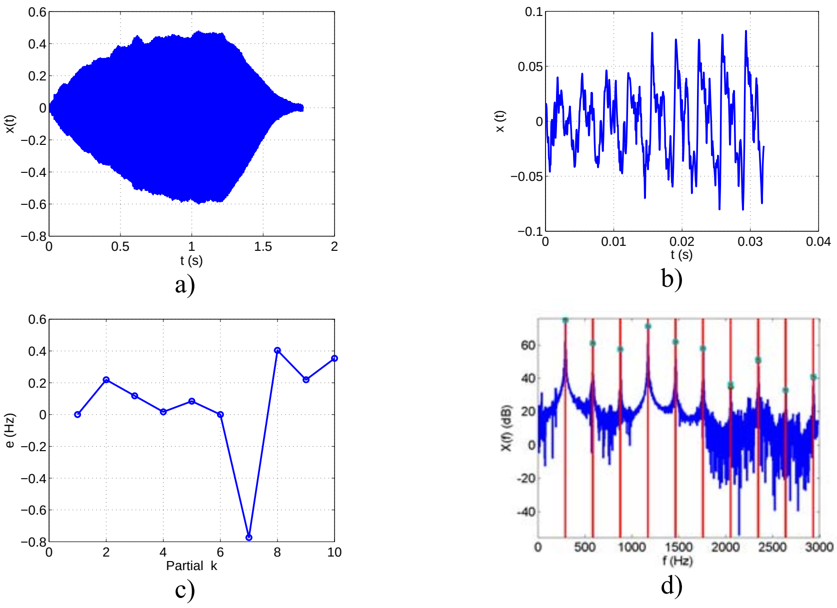


Fig. 4. Signal of the tone D4 produced by playing with the fiddle-bow.

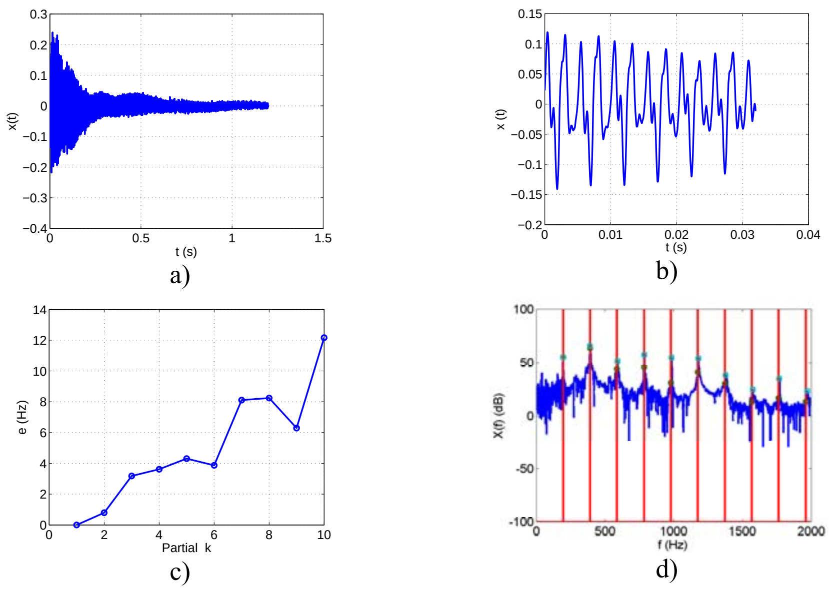


Fig. 5. Signal of the tone G3 produced by picking the string..

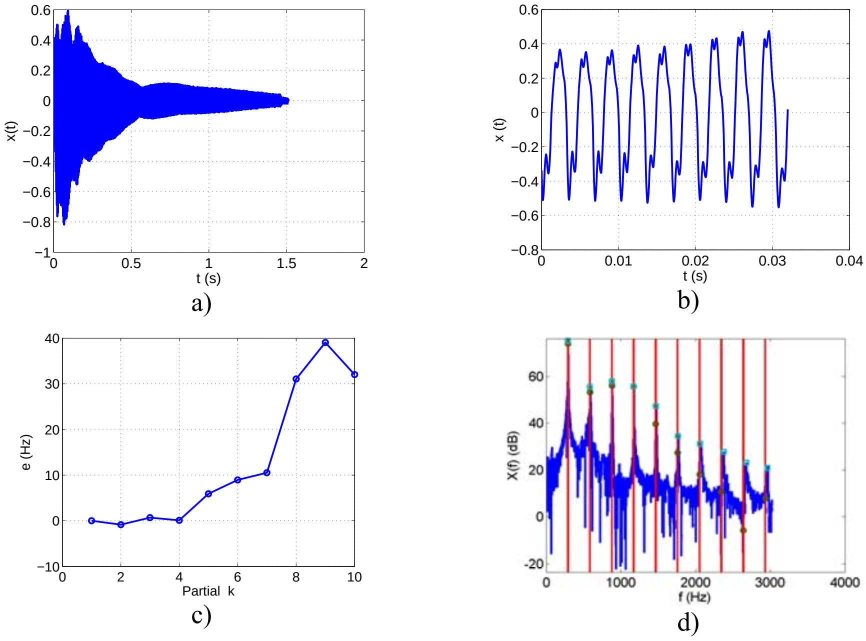


Fig. 6. Signal of the tone D4 produced by picking the string.

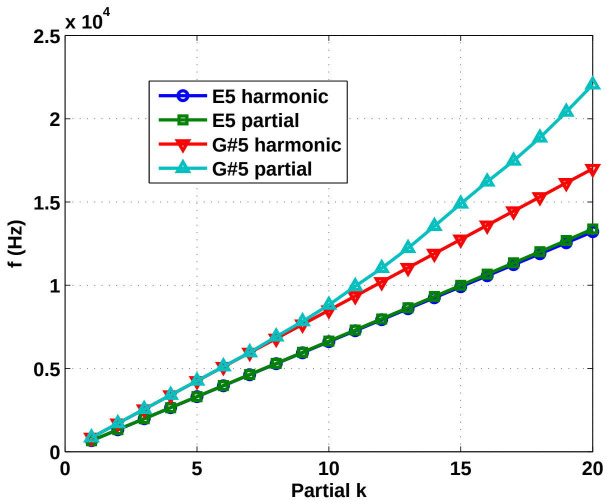


Fig. 7. Frequential position of aliquote components (harmonics and inharmonious) when playing (E5,G#5).

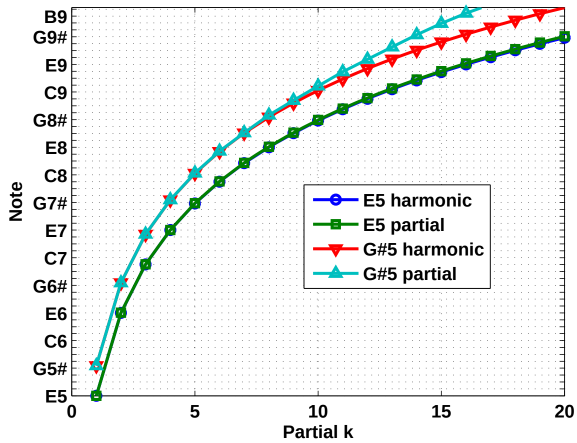
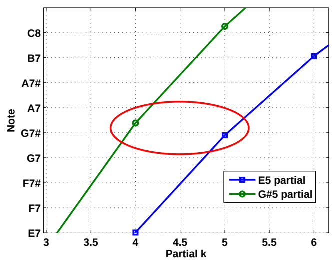
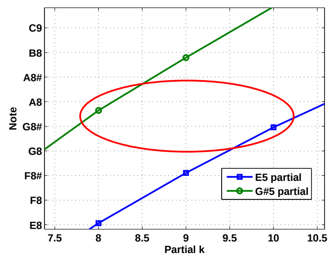


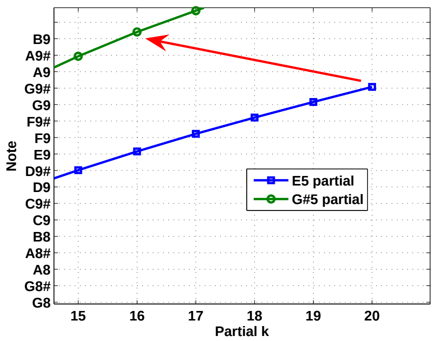
Fig. 8. Note position of aliquote components (harmonics and inharmonious) when playing (E5,G#5).



a)



b)



c)

Fig. 9. Inharmonious aliquotes position details with E5-G#5: a) G#7, b) G#8, and c) G#7.

Table I
Coefficient of inharmonicity of tones of the empty string of the violin.

	F0 [Hz]	Fiddle-bow	Pizzicato
		β (x10^{-6})	β (x10^{-4})
G3	196	6.05	1.31
D4	293.66	1.38	2.86
A4	440	3.55	3.82
E5	659.25	2.06	1.13
		βsr=3.26	**βsr=2.28**

Table II
Coefficient of inharmonicity of tones of the first and second string of the violin.

Tone	β_S (x10^{-4})
E5	1.13
F5	6.9341
Fis5	6.8746
G5	2.5016
Gis5	6.1697
A5	18.2261
$\overline{\beta}_P$	6.9726

Table III
Coefficient of inharmonicity of tones of the empty string of the violin.

Tone	MAFE [Hz]	MACE [cent]
E5-F5	413.0169	63.6348
E5-Fis5	654.7776	101.5170
E5-G5	93.9050	12.8624
E5-Gis5	1067.6	150.2831
E5-A5	276.6344	64.3648
	$\overline{MAFE} = 501.1867$	$\overline{MACE} = 78.5324$

C. Analysis of the results

On the base of the results shown in Fig. 3 through Fig. 9 and in Tab. I-III it can be concluded that:

a) the coefficient of inharmonicity at the pizzicato technique is 2.28x10^{-4}/3.26x10^{-6}=69.9 times greater than the one obtained when the string was aroused by playing with the fiddle-bow,

b) values of the coefficient of inharmonicity of the sound produced by the fiddle-bow is of order 10^{-6} and can be considered that thare is no inharmonicity

c) when the pizzicato techhnique is used, inharmonicity of the violin is aproximately in the same limits as at the pianino August Förster (1÷2)x10^{-4} [11] and the electric guitar

Stratokaster (0.8÷2.5)x10^{-4} [6], while it is greater for the size order at the concert piano Steinway D (0.6÷0.8)x10^{-4} [10].

d) analysis of the double stops produced by the pizzicato technique on the first and second strings showed that the mean inharmonicity error was $\overline{MAFE} = 501.1867$ Hz, while the mean cent error was $\overline{MACE} = 78.5324$ cent. In comparison to the results of inharmonicity of the double stops of the pianino "August Förster" in the counter octave ($\overline{MACE} = 31.8357$ cent) [12] the analysed violin obviously had 2.46 times greater cent error.

The mentioned results point to the fact that at instruments where the sound is produced by picking, their inharmonicity is considerable because of irregular oscillating of the string after the arousal when it is let to oscillate freely. On the other hand, when aroused by a fiddle-bow, it is not oscillating freely and inharmonicity is negligible

Considering the fact that on the cent scale the distance between any two adjacent halftones is 100 cent, it can be concluded that the mean cent error is 78% of a halftone. In Fig. 4 it can be seen that considerable aberrations of the inharmonic components in relation to the harmonic ones for the analysed double stops occur above 10 Hz. In this range man has lesser sensitivity. Reviewing all the presented results it can be said that the analysed violin has good acoustic parameters, which is confirmed by its price on the market.

Further investigations will be directed toward testing of greater number of instruments from the group of string instruments (that can be played by picking or by using the fiddle-bow) and according to the results hopefully a global conclusion will be made

4. CONCLUSION

This paper has presented the results of testing the copy of Antonius Stradivarius violin built in the Czeck Republic in the transition period from the 19^{th} to the 20^{th} century, regarding inharmonicity. The testing was performed for two ways of producing the sound: by picking the string (the pizzicato technique) or by playing with a fiddle-bow. The results show that inharmonicity is negligible if the fiddle-bow is used (the coefficient of inharmonicity of order 10^{-6}), while if it the pizzicato technique is used the coefficient of inharmonicityis of order 10^{-4}. The mean value of inharmonicity of the double stops of the violin in the second octave is $\overline{MACE} = 78.5324$ cent, which is 2.46 times higher in comparison to inharmonicity of the double sounds of "August Förster" for the double stops from the counter octave. Considering the fact that the spectral components of tones from the second octave are in higher parts of the spectrum, where man has lesser sensitivity, it can be said that inharmonicity of the violin played with the pizzicato technique is of the same order of magnitude as of the pianino "August Förster" and the electric guitar "Fender Stratokaster".

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