COMPARISON OF THE ALGORITHMS FOR CS IMAGE RECONSTRUCTION

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Abstract: This paper describes comparison of algorithms for Compressive Sensing reconstruction of 2D signals. Compressive Sensing is a new signal sensing approach aiming to decrease the requirements for resources in real digital systems (number of sensors, memory requirements, etc.). This method provides signal analysis and reconstruction using small set of randomly chosen samples. Reconstruction is based on complex mathematical algorithms - optimization algorithms. Depending on the signal type, different optimization algorithms are used. This paper deals with three algorithms for CS image reconstruction. Performances of the algorithms are compared for different types of 2D signals. Reconstruction quality is measured by calculating PSNR between original and reconstructed signal. Execution time for each considered algorithm is calculated, as well.

1. INTRODUCTION

Standard approach to signal processing is based on the signal sampling according to the Shannon-Nyquist theorem. Signal has to be sampled with frequency which is at least two times higher than the maximal signal frequency, in order to provide high accuracy signal reconstruction. This way of sampling requires significant resources for storing and transmitting data, and therefore, signal has to be compressed. In recent years, there is an intensive growth of the alternative ways for signal sampling, based on Compressive Sensing (CS) methods [1]-[5]. CS is new method for signal acquisition, based on sampling with frequency beyond Nyquist. CS provides high accuracy signal reconstruction, despite the reduction of number of signal samples. However, certain conditions have to be satisfied in order to apply CS reconstruction method. Namely, signal has to be sparse in its own or in some transform domain (discrete Fourier transform domain, discrete cosine transform domain, wavelet domain, etc.). Another condition that has to be satisfied in CS is related to the sampling/measurement procedure. Measurement procedure has to provide signal reconstruction from small number of acquired samples called measurements. Reconstruction from the small number of samples requires powerful mathematical solvers to be used – i.e. optimization algorithms. A large number of optimization algorithms are

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used nowadays, and they are still intensively developing. Commonly used are optimization algorithms based on ℓ_1-norm minimization.

The possibility of 2D under-sampled signal reconstruction using CS is analyzed in this paper [1]-[5], [11]-[13]. Two types of 2D signals are considered - the real life, natural images and images used in medical purposes. The majority of 2D signals do not satisfy the first CS condition – sparsity. Therefore, the alternative CS algorithms are developed for reconstruction of the signals which are not perfectly sparse. One of them is Total Variation method (TV) [1]. TV is based on signal gradient ℓ_1-norm minimization. TV uses gradient minimization, as gradient of 2D signals has sparse representation in its own or in some transform domain.

Different algorithms for the 2D signal reconstruction are considered in this paper. All of the described algorithms are based on TV minimization. The comparison of the reconstruction quality for different number of measurements is given, as well as the algorithm execution time.

The paper is organized as follows. The theoretical background on the intensively studied CS methods for signal processing, their main properties, as well as the procedure for CS signal acquisition, is given in Section II. The algorithms used in this paper are described in the Section III, while the experimental results along with the discussion could be found in the Section IV. The concluding remarks are given in the Section V.

2. COMPRESSIVE SENSING

Finite, real, 1D signal x could be described as column, $N\times 1$ vector, in the R^N space. Mathematically, signal could be described as [1]-[4]:

$$x = \sum_{i=1}^{N} s_i \psi_i = \psi s, \tag{1}$$

where ψ denotes $N\times N$ transform domain matrix, while s is transform coefficients vector. Vectors x and s are representations of the same signal in different domains - in time (space) domain and in transform domain (denoted with ψ in this paper). Two conditions have to be satisfied in order to reconstruct signal from the small number of acquired samples. First, signal has to be sparse, which means that small number of signal coefficients (in its own domain or in the transform domain) has non-zero values. The second condition is related to the measurement procedure. Namely, measurement procedure has to be incoherent. If the incoherence property is satisfied, the signal will be reconstructed with high accuracy using small number of acquired samples. The majority of real signals satisfy sparsity property and the measurement procedure could be performed in a way which satisfies incoherence condition (by randomly acquiring signal samples).

The number of acquired signal samples (i.e. measurements) can be much smaller than the signal length N, i.e. $M<<N$. Measurement vector y is obtained by multiplication of the measurement matrix ϕ by signal vector x, which could be described as [1], [11], [12]:

$$y = \phi x. \tag{2}$$

Combining the relations (1) and (2) the following equation is obtained:

$$y = \phi x = \phi \psi s = \theta s \,. \tag{3}$$

Matrix $\theta = \phi\psi$ denotes $M \times N$ Compressive Sensing matrix. The procedure for samples acquisition is simple, while the reconstruction procedure requires complex mathematical algorithms. System of equations (3) is undetermined system and has infinite number of solutions. In order to obtain unique solution, the optimization algorithms are used. One of the commonly used is optimization based on the ℓ_1-norm minimization:

$$s = \min \| s \|_{l_1} \; subject \; to \; y = \theta s \,. \tag{4}$$

3. ALGORITHMS FOR CS IMAGE RECONSTRUCTION

Traditional way of signal acquisition is based on Shannon-Nyquist sampling theorem. According to this theorem, signal will be reconstructed with high accuracy if it is sampled at frequency which is twice the maximal signal frequency. Sampling in such way results in large number of signal samples. Therefore, signal has to be compressed in order to be further processed. CS is a method that performs acquisition and compression at the same time. This saves memory and shortens signal acquisition time. Additionally, CS enables reconstruction and processing of the signals in the cases when missing samples occur (which is common case in the real applications).

As it was mentioned before, the signal can be reconstructed from its measurement using complex optimization algorithms. Commonly used optimization is based on ℓ_1 minimization. Generally speaking, image is not sparse in any transform domain and hence the reconstruction quality obtained with ℓ_1 minimization will not produce satisfactory results. Therefore, ℓ_1 minimization of the signal gradient (TV minimization) is used for the 2D signal reconstruction [1], [12], [13].

TV of the signal *s* is sum of the gradient amplitudes in the point *(i,j)*. It can be described as:

$$\|s\|_{TV} = \sum_{i,j} \left|(\nabla s)_{ij}\right|, \tag{5}$$

where ∇ represents differentiation operator, i.e. approximate value of the gradient, for pixel *(i,j)*:

$$\nabla_{i,j} s = \begin{bmatrix} s(i+1,j) - s(i,j) \\ s(i,j+1) - s(i,j) \end{bmatrix} \tag{6}$$

Discrete form of the TV could be described as:

$$TV(s) = \sum_{i,j} \sqrt{(s_{i+1,j} - s_{i,j})^2 + (s_{i,j+1} - s_{i,j})^2} \tag{7}$$

TV minimization problem is given by equation (8):

$$\min TV\ (s)\ \ subject\ \ to\ \ y\ =\ \theta s \tag{8}$$

TV minimization provides a reliable reconstruction of the signal and gives satisfactory results in the cases of noisy signals also. In the sequel, the algorithms that are used for the 2D signals reconstruction in this paper are described. All of the described algorithms take measurements from the frequency domain of signal, and obtain solution by using TV minimization.

Algorithm 1:

The first algorithm described in the paper takes measurements form the 2D Fourier transform of the signal. The samples are taken by using radial line mask (Fig. 1) [13]. The number of radial lines in the mask could be changed and consequently, the number of taken measurements is changed. Samples taken from the mask form measurement vector which is used in the TV based reconstruction procedure.

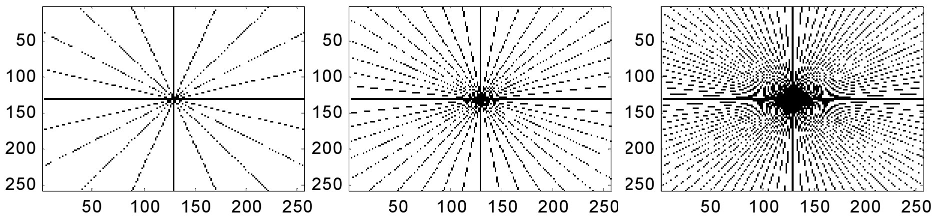

Fig. 1: Radial line mask (L=10, L=22 and L=50, from left to right)

Algorithm 2:

The second algorithm for image reconstruction is based on taking measurements from the 2D Discrete Cosine Transform (2D DCT), in a random manner. The image could be treated as a whole, or can be divided into blocks. Measurements are randomly taken from image or from each image block separately, in order to form measurement vector. In the case of block-separated image, the reconstruction is performed block by block. The reconstruction quality depends on the block size, as well as on the number of samples taken form the block (or from the whole image).

Algorithm 3:

Third algorithm deals with the 2D DCT coefficients taken as measurements. This algorithm takes certain number of low frequency (LF) coefficients in measurement procedure, to assure good reconstruction quality. This number LF coefficients could be changed, depending on required reconstruction quality. The algorithm requires that all of 2D DCT image coefficients are known, in order to choose the LF ones. This could be limiting factor for the algorithm application in cases when there is no information about all image coefficients.

Beside LF coefficients, the algorithm takes certain number of middle or high frequency coefficients (MF, HF), as well [11]. The image (or image block) is firstly converted into a vector by zigzag rearrangement. Then the measurement vector is formed of K_1 LF coefficients and K_2 randomly selected MF or HF coefficients.

The measurement vector is described with the relation $y=y_1+y_2$, where y_1 denotes LF coefficients vector and y_2 denotes MF and/or HF coefficients vector. The 2D signal is reconstructed from its measurements, by using relation (7).

In all considered cases, the reconstruction quality is measured by peak signal to noise ratio (PSNR), defined as:

$$PSNR = 20\log_{10} \frac{O_{\max}}{\sqrt{\frac{1}{MN}\sum_{i=0}^{M-1}\sum_{j=0}^{N-1}[S_{orig}(i,j)-S_{rec}(i,j)]^2}}, \tag{9}$$

where O_{max} denotes maximum luminance in the image, M and N are image size, S_{orig} and S_{rec} denote original and reconstructed image.

4. EXPERIMENTAL RESULTS

The experimental results for the reconstruction of the several test images are given in this section. The „Cameraman“ image as well as two medical images („Brain“ and „Phantom“) are considered. The images are of 256x256 size and their original versions are shown in Fig. 2.

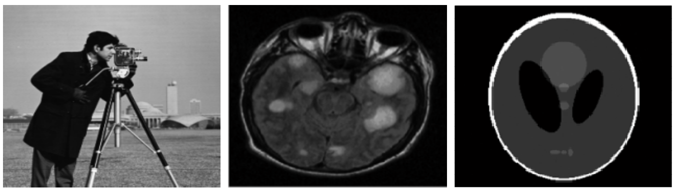

Fig. 2: Original Cameraman, MRI Brain and Phantom images

A. Algorithm 1 reconstruction

The measurements are taken from the DFT domain by using radial line mask. The number of lines in the mask is user defined.

Table 1: Simulation results for the Algorithm 1

Phantom image			Cameraman image		
Number of lines in radial line mask	**Time (s)**	**PSNR (dB)**	**Number of lines in radial line mask**	**Time (s)**	**PSNR (dB)**
100	163.94	121.48	100	1078.8	33.21
22	577.69	48.80	22	1123.2	22.68
15	578.28	34.47	15	1046,8	21.56
10	505.45	20.14	10	1102.9	19.85

Fig. 3: “Phantom” and “Cameraman” reconstruction using different number of lines in DFT domain

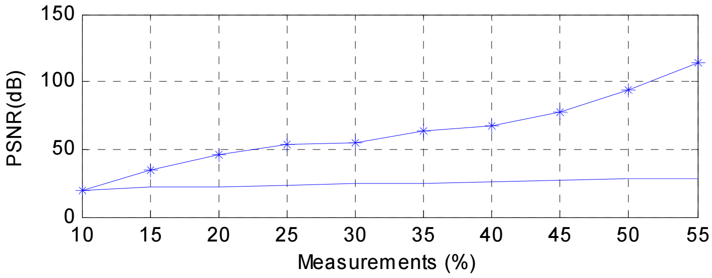

Fig. 4: PSNR for different number of radial lines in DFT domain; solid line is for “Cameraman” image and solid-star line is for “Phantom” image

In the simulations, the number of lines is changed and PSNR is calculated for each mask.

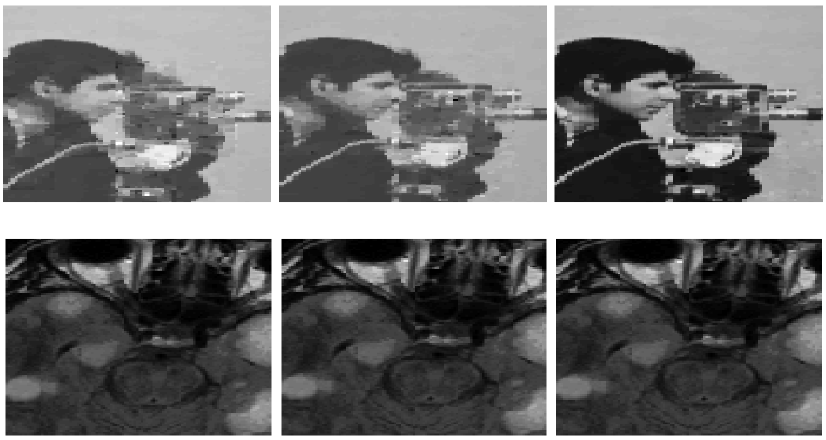

Fig. 5: Reconstruction results using Algorithm 2, with block sizes 16x16, 32x32 and 64x64, respectively (from left to right). Results are given for 35% of measurements per block

Fig. 3 shows reconstructed images, using number of measurements defined in the Table 1 (for number of radial lines L=10, L=22, L=100, respectively).The medical images, like

“Brain” and “Phantom”, require smaller number of lines to be used when defining the mask. It is shown that L=22 provides reconstruction with PSNR=48.8031 dB, which indicates good image quality. Smaller number of lines means smaller number of samples used for reconstruction, and thus lower reconstruction time. Low reconstruction time is especially important in medicine (e.g. Magnetic Resonance Imaging), where patient is exposed to dangerous waves for certain amount of time. Fig. 4 shows PSNR values, using different number of radial lines in the mask, for two different types of images – natural and medical image. For real, natural images reconstruction (like “Cameraman” image), the number of lines has to be much larger (L=100), in order to obtain satisfactory reconstruction, i.e. PSNR>30 dB.

B. Algorithm 2 reconstruction

The satisfactory reconstruction quality can be obtained by taking samples from the 2D DCT domain and using TV minimization. In order to improve reconstruction quality, the image is divided into blocks. Different block sizes are considered. Also, the number of measurements per block, for the fixed block size, is also changed, in order to test reconstruction quality. The 16x16, 32x32, 64x64 block sizes are observed. The 35% and 70% of measurements from each block are taken (and for each block size). Algorithm execution time is measured, as well as PSNR between original and reconstructed image. The results are given in Table 2. Fig. 5 shows reconstructed images, using three different block sizes: 16x16, 32x32 and 64x64. The larger block size provides better reconstruction results (for the same percentage of measurements as in smaller blocks). This can be explained with the fact that longer signals have better sparsity compared to the short ones. Fig. 6 shows PSNR-number of measurements graph, for different block sizes and for different images. As it can be seen, PSNR is larger for medical image, in all considered cases.

Table 2: Simulation results for the Algorithm 2

Block size	**Number of measurement (%)**	**Time (s)**	**PSNR (dB)**	**Block size**	**Number of measurement (%)**	**Time (s)**	**PSNR (dB)**
Cameraman image				**MRI Brain image**			
16x16	35	235.09	26.9894	16x16	35	233.76	32.5143
32x32		115.17	28.2634	32x32		108.16	34.4819
64x64		80.33	30.1655	64x64		83.50	35.6922
16x16	70	154.20	33.8066	16x16	70	185.03	40.3012
32x32		72.74	35.2401	32x32		93.33	41.9663
64x64		63.82	37.1667	64x64		71.29	43.4082

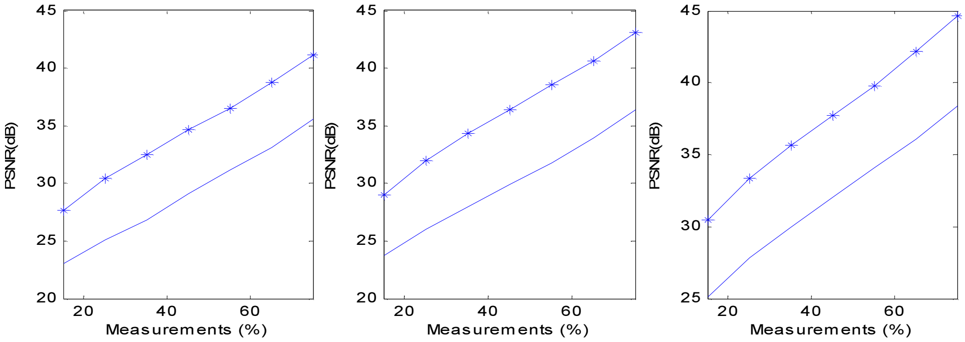

Fig. 6: PSNR dependence on the number of measurement. Solid line is for the "Cameraman" image and solid-star line is for the "Brain" image. Results are for 16x16, 32x32 and 64x64 block sizes (from left to right)

C. *Algorithm 3 reconstruction*

Table 3 shows parameters used in reconstruction with Algorithm 3, where K_1 denotes number of LF coefficients and K_2 denotes number of HF coefficients. Percentage of taken measurements, reconstruction quality measure – PSNR and algorithm execution time are given in the Table 3, as well. The test images are of 64x64 size. In all considered cases, reconstruction using 20%, 30%, 40% and 50% measurements is tested. Different number of LF coefficients is used: K_1=100, K_1=500 and K_1=800. For each number K_1, the number K_2 of HF coefficients is chosen such that the sum K_1+K_2 is equal to : a) 20%, b) 30%, c) 40% and d) 50% of the total number of coefficients. Higher number of LF coefficients requires higher execution time, for the same total number of measurements (see Table 3). Also, the more LF coefficients is used (for the same total number of measurements), the better PSNR is achieved. The natural, „Cameraman“ image, requires 50% of coefficients to be used for the reconstruction with PSNR=30 dB. The same reconstruction quality in the case of medical image can be obtained using 20% of the coefficients.

Table 3: Simulation results obtained by using Algorithm 3

Image (64x64)	**Meas. (%)**	**K1 LF**	**K2 HF**	**Time (s)**	**PSNR (dB)**	**Image (64x64)**	**Meas. (%)**	**K1 LF**	**K2 HF**	**Time (s)**	**PSNR (dB)**
Cameraman	20%	100	720	6.10	20.2051	Brain	20%	100	720	4.71	28.3956
		500	320	6.36	21.4185			500	320	5.14	29.6162
		800	20	6.06	21.8561			800	20	6.46	30.1297
	30%	100	1130	8.12	22.6839		30%	100	1130	4.19	31.1564
		500	730	9.09	23.1146			500	730	5.23	32.3775
		800	430	11.24	23.7023			800	430	5.42	32.7475
	40%	100	1530	7.9	24.9610		40%	100	1530	4.22	33.4567
		500	1130	9.26	26.2603			500	1130	5.11	34.2337
		800	730	10.73	25.67			800	730	6.15	34.1546
	50%	100	1950	7.08	28.5986		50%	100	1950	3.87	35.8391
		500	1550	10.99	30.1110			500	1550	5.32	36.3860
		800	1250	11.48	30.2483			800	1250	6.99	37.4498

Fig. 7: „Cameraman" reconstruction using different LF-HF ratios. The percentage of measurements from the total number of image coefficients is: 20% for the first row, 30% for the second row, 40% for the third row and 50% for the forth row. The LF=100, 500 and 800 is used for the pictures in one row, from left to right

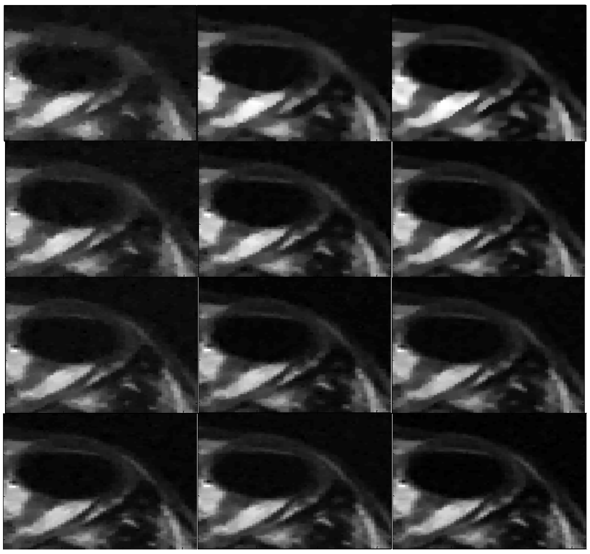

Fig. 8: „Cameraman" reconstruction using different LF-HF ratios. The percentage of measurements from the total number of image coefficients is: 20% for the first row, 30% for the second row, 40% for the third row and 50% for the forth row. The LF=100, 500 and 800 is used for the pictures in one row, from left to right

Figs 7 and 8 show reconstructed images with the parameters in accordance with the Table 3. The lack of the algorithm is the necessity of LF coefficients presence in the measurement vector. This means that, during signal acquisition, these coefficients must be recorded and stored.

5. CONCLUSION

Different algorithms for CS image reconstruction are considered, analysed and discussed in this paper. Due to the specific nature of images, the method based on gradient minimization, i.e. Total Variation minimization, is used for reconstruction in all considered cases. The measurements are taken from the frequency domain in each observed algorithm, but the way of coefficient selection is different for considered approaches. The reconstruction quality, as well as execution time, is compared for different algorithms and different number of measurements. It is shown that the medical images can be reconstructed with smaller number of measurements, for all described algorithms. This type of images are more sparse in the certain transform domain. Therefore, good reconstruction quality could be obtained with less measurements, compared to the natural images. The best results are obtained in cases when we can chose certain number of LF coefficients in measurement vector, as it was done in the last described algorithm. However, this could be limiting factor, for example, in the cases when the missing pixels occurs. In the cases when the complete information about the image is available, this algorithm gives satisfactory reconstruction results and decreases the processing time (compared with the case when we deal with the full set of image samples).

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